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Two-Step Hybrid Block Method for Solving First Order Ordinary Differential Equations Using Power Series Approach

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Authors' contributions

This work was carried out in collaboration between all authors. Author GA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author SAA managed the analyses of the study. Author ODO managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper, we consider the derivation of hybrid block method for the solution of general first order Initial Value Problem (IVP) in Ordinary Differential Equation. We adopted the method of Collocation and Interpolation using power series approximation to generate the continuous formula. The properties and features of the methods are analyzed and some numerical examples are also presented to illustrate the accuracy and effectiveness of the method.

Keywords: Collocation; interpolation; linear multistep method; hybrid and power series polynomial.

1 Introduction

In recent times, the integration of Ordinary Differential Equations (ODEs) is carried out using block methods. In this paper, we propose an order five hybrid block integrator method for the solution of first order ODEs of the form:

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$$y' = f(x, y),\ y(a) = y_o,\quad x\epsilon[a.b] \tag{1.0}$$

Where f is continuous within the interval of integration$[a, b]$. We assume that f satisfies Lipchitz condition which guarantees the existence and uniqueness of solution of (1.0).The discrete solution of (1.0) using linear multistep method has being studied by authors like [1] and continuous solution of (1.0) [2] and [3,4]. One important advantage of the continuous over discrete approach is the ability to provide discrete schemes for simultaneous integration. These discrete schemes can be reformulated to general linear methods (GLM) [5]. The block methods are self-starting and can be applied to both stiff and non-stiff initial value problem in differential equations. More recently, authors like [6,7,8,9] and [10] to mention few proposed methods ranging from predictor- corrector to hybrid block method for initial value problem in ordinary differential equation.

In this work, hybrid block method using Power series expansion will be considered. This will help in coming up with a more computationally reliable integrator that can solve first order differential equations problems of the form (1.0).

2 Derivation of Hybrid Method

In this section, we intend to construct the proposed two-step linear multistep method which will be used to generate the main method and other methods required to set up the block method. We consider the power series polynomial of the form:

$$P(x) = \sum_{j=0}^{n} a_j x^j \tag{2.0}$$

which is used as our basis to produce an approximate solution to (1.0) as

$$y(x) = \sum_{j=0}^{m+t-1} a_j x^j \tag{3.0}$$

and

$$y'(x) = \sum_{j=0}^{m+t-1} j a_j x^{j-1} = f(x, y) \tag{4.0}$$

where a_j are the parameters to be determined, m and t are the points of collocation and interpolation respectively. This process leads to $(m + t - 1)$ of non-linear system of equations with unknown coefficients, which are to be determined by the use of Maple 17 Mathematical software.

3 Hybrid Block Method

Using equation (3.0) and (4.0), m=1 and t=5 our choice of degree of polynomial is $(m + t - 1)$. Equation (3.0) is interpolated at the point $x = x_n$ and equation (4.0) is collocated at $x = (0, \frac{1}{2}, 1, \frac{3}{2}, 2)$ which lead to system of equation of the form

$$\sum_{j=0}^{m+t-1} a_j x_{n+i}^j = y_{n+i} \quad i = 0 \tag{5.0}$$

$$\sum_{j=0}^{m+t-1} j a_j x_{n+i}^j = f_{n+i} \quad i = (0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2) \tag{6.0}$$

With the mathematical software, we obtained the continuous formulation of equations (5.0) and (6.0) of the form

$$y(x) = \alpha_0 y_n + h[\beta_0 f_n + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} + \beta_1 f_{n+1} + \beta_{\frac{3}{2}} f_{n+\frac{3}{2}} + \beta_2 f_{n+2}] \tag{7.0}$$

After obtaining the values of α_j and β_i, $j = 0$ and $i = (0, \frac{1}{2}, 1\frac{3}{2}, 2)$ in (7.0)

We evaluated at the point $x = x_{n+j}$, $j = (1, \frac{1}{2}, \frac{3}{2}, 2)$ which gives the following set of discrete schemes to form our hybrid block method.

$$\left.\begin{aligned}
y_{n+1} &= y_n + \frac{29}{180}hf_n + \frac{31}{45}hf_{n+1/2} + \frac{2}{15}hf_{n+1} + \frac{1}{45}hf_{n+3/2} - \frac{1}{180}hf_{n+2} \\
y_{n+1/2} &= y_n + \frac{251}{1440}hf_n + \frac{323}{720}hf_{n+1/2} - \frac{11}{60}hf_{n+1} + \frac{53}{720}hf_{n+3/2} - \frac{19}{1440}hf_{n+2} \\
y_{n+3/2} &= y_n + \frac{27}{160}hf_n + \frac{51}{80}hf_{n+1/2} + \frac{9}{20}hf_{n+1} + \frac{21}{80}hf_{n+3/2} - \frac{3}{160}hf_{n+2} \\
y_{n+2} &= y_n + \frac{7}{45}hf_n + \frac{32}{45}hf_{n+1/2} + \frac{4}{15}hf_{n+1} + \frac{32}{45}hf_{n+3/2} + \frac{7}{45}hf_{n+2}
\end{aligned}\right\} \quad (8.0)$$

Equations (8.0) are of uniform order five, with error constant as follows

$$[\frac{1}{5760}, \frac{3}{10240}, \frac{3}{10240}, -\frac{1}{15120}]^T$$

4 Consistency

Definition: The Linear Multistep method is said to be consistent if it is of order P≥ 1 and its first and second characteristic polynomial defined as $\rho(z) = \sum_{j=0}^{k} \alpha_j z^j$ and $\sigma(z) = \sum_{j=0}^{k} \beta_j z^j$ where Z satisfies
$(i) \sum_{j=0}^{k} \alpha_j = 0, (ii) \rho'(1) = 0, (iii)\ \rho''(1) = 2!\, \sigma(1)$, See [1].

The discrete Schemes derived are all of order greater than one and satisfy the condition (i)-(iii).

5 Zero Stability of the Block Method

The block method is defined by [11] as

$$Y_m = \sum_{i=0}^{k} A_i + h \sum_{i=0}^{k} B_i F_{m-i}$$

$$\text{where } Y_m = [y_n, y_{n+1}, y_{n+2}, \dots, y_{n+r-1}]^{\mathrm{T}}$$
$$F_m = [f_n, f_{n+1}, f_{n+2}, \dots, f_{n+r-1}]^{\mathrm{T}}$$

$A_i's$ and $B_i's$ are chosen r x r matrix coefficient and $m = 0,1,2$ ... represents the block number, $n = mr$, the first step number in the m-th block and r is the proposed block size.

The block method is said to be zero stable if the roots of $R_j, j = 1(1)k$ of the first characteristics polynomial is

$$\rho(R) = \det\left[\sum_{i=0}^{k} A_i R^{k-1}\right] = 0, A_0 = I$$

satisfies $|\mathbf{R_j}| \leq \mathbf{1}$, if one of the roots is +1, then the root is called Principal Root of $\rho(R)$.

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+1}\\y_{n+1/2}\\y_{n+3/2}\\y_{n+2}\end{bmatrix}=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-3/2}\\y_{n-1/2}\\y_{n-1}\\y_{n}\end{bmatrix}+h\begin{bmatrix}\frac{31}{45}&\frac{2}{15}&\frac{1}{45}&\frac{1}{180}\\\frac{323}{720}&\frac{-11}{60}&\frac{53}{720}&\frac{-19}{1440}\\\frac{51}{80}&\frac{9}{20}&\frac{21}{80}&\frac{-3}{160}\\\frac{32}{45}&\frac{4}{15}&\frac{32}{45}&\frac{7}{45}\end{bmatrix}\begin{bmatrix}f_{n+1/2}\\f_{n+1}\\f_{n+3/2}\\f_{n+2}\end{bmatrix}$$

$$+\begin{bmatrix}0&0&0&\frac{29}{180}\\0&0&0&\frac{251}{1440}\\0&0&0&\frac{27}{160}\\0&0&0&\frac{7}{45}\end{bmatrix}\begin{bmatrix}f_{n-3/2}\\f_{n-1}\\f_{n-1/2}\\f_{n}\end{bmatrix}$$

where

$$A^{(0)}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ A^{(1)}=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix},\ B^{(0)}=\begin{bmatrix}\frac{31}{45}&\frac{2}{15}&\frac{1}{45}&\frac{1}{180}\\\frac{323}{720}&\frac{-11}{60}&\frac{53}{720}&\frac{-19}{1440}\\\frac{51}{80}&\frac{9}{20}&\frac{21}{80}&\frac{-3}{160}\\\frac{32}{45}&\frac{4}{15}&\frac{32}{45}&\frac{7}{45}\end{bmatrix}\text{ and }B^{(0)}=\begin{bmatrix}0&0&0&\frac{29}{180}\\0&0&0&\frac{251}{1440}\\0&0&0&\frac{27}{160}\\0&0&0&\frac{7}{45}\end{bmatrix}$$

The first characteristics polynomial of the scheme is

$$\rho(\lambda)=\det\,[\lambda A^{0}-A^{1}]$$

$$\rho(\lambda)=det\left[\begin{pmatrix}\lambda&0&0&0\\0&\lambda&0&0\\0&0&\lambda&0\\0&0&0&\lambda\end{pmatrix}-\begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{pmatrix}\right]$$

$$\rho(\lambda)=det\begin{bmatrix}\lambda&0&0&-1\\0&\lambda&0&-1\\0&0&\lambda&-1\\0&0&0&\lambda-1\end{bmatrix}$$

$$\begin{vmatrix}\lambda&0&0&-1\\0&\lambda&0&-1\\0&0&\lambda&-1\\0&0&0&\lambda-1\end{vmatrix}=0$$

$$\lambda^{3}(\lambda-1)=0$$

$\lambda_1 = \lambda_2 = \lambda_3 = 0 \; or \; \lambda_4 = 1$

We can see clearly that no root has modulus greater than one (i.e. $\lambda_i \leq 1$) $\forall i$. The hybrid block method is zero stable.

6 Numerical Examples

Problem 1:

$y' = y, \quad y(0) = 1, h = 0.1$

Exact Solution: $y(x) = \exp(x)$

Table 1. Comparison of approximate solution of problem 1

x	Exact solution	Proposed scheme	Error in proposed scheme	Error in [2]
0.1	1.105170918075648	1.105170917860730	2.149179E-10	1.226221039551945e-05
0.2	1.221402758160170	1.221402757685120	4.7505E-10	1.355183832019158e-05
0.3	1.349858807576003	1.349858806788490	7.875129E-10	1.497709759790133e-05
0.4	1.491824697641270	1.491824696480820	1.16045E-09	1.655225270247307e-05
0.5	1.648721270700128	1.648721269097010	1.603118E-09	1.829306831546695e-05
0.6	1.822118800390509	1.822118798264440	2.126069E-09	2.021696710463594e-05
0.7	2.013752707470477	2.013752704729200	2.741277E-09	2.234320409577606e-05
0.8	2.225540928492468	2.225540925030090	5.989459E-09	2.469305938346267e-05
0.9	2.459603111156950	2.459603106852120	4.30483-09	2.729005110868599e-05
1.0	2.718281828459046	2.718281824122030	4.337016E-09	3.01601708376864e-05

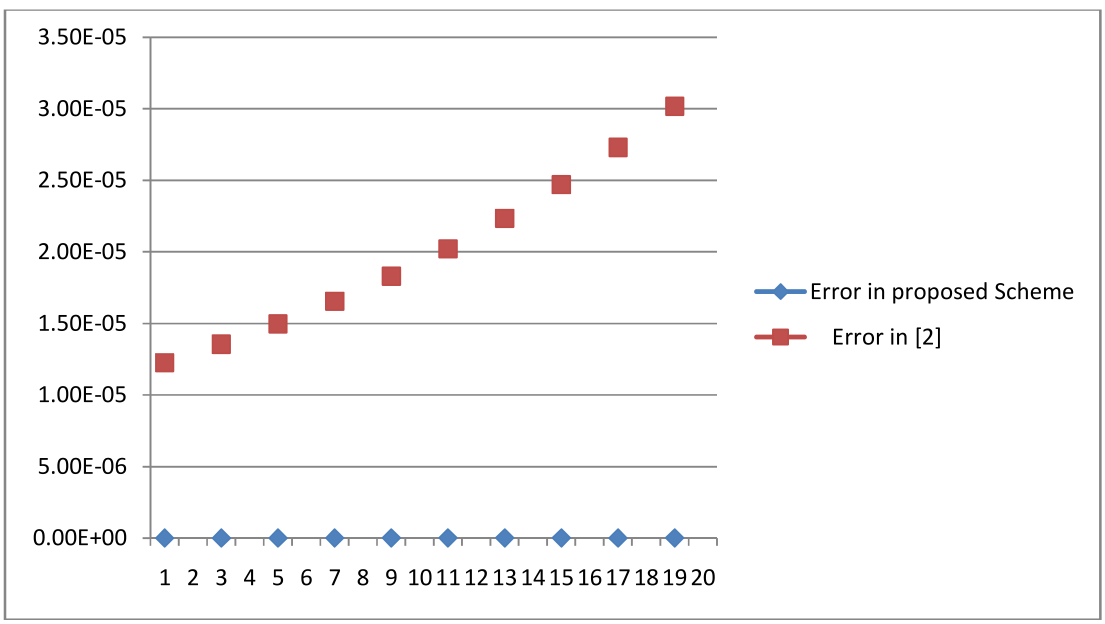

Fig. 1. Plot of error in proposed scheme and error in [2]

Problem 2:

$y' = 0.5(1 - y), \quad y(0) = 0.5, h = 0.1$

Exact Solution: $y(x) = 1 - 0.5\mathrm{e}\ -0.5\mathrm{x}$

Table 2. Comparison of approximate solution of problem 2

x	Exact solution	Proposed scheme	Error in proposed scheme	Error in [7]
0.1	0.524385287749643	0.524385287750861	1.218026E-13	5.574430e-012
0.2	0.547581290982020	0.547581290981880	1.399991E-13	3.946177e-012
0.3	0.569646011787471	0.569646011786286	1.184941E-12	8.183232e-012
0.4	0.590634623461009	0.590634623462548	1.538991E-12	3.436118e-011
0.5	0.610599608464297	0.610599608463187	1.110001E-12	1.929743e-010
0.6	0.629590889659141	0.629590889658614	5.270229E-12	1.879040e-010
0.7	0.647655955140643	0.647655955142752	2.10898E-12	1.776835e-010
0.8	0.664839976982180	0.664839976969201	1.297895E-11	1.724676e-010
0.9	0.681185924189113	0.681185924158290	3.08229E-11	1.847545e-010
1.0	0.696734670143683	0.696734670139561	4.121925E-11	3.005770e-010

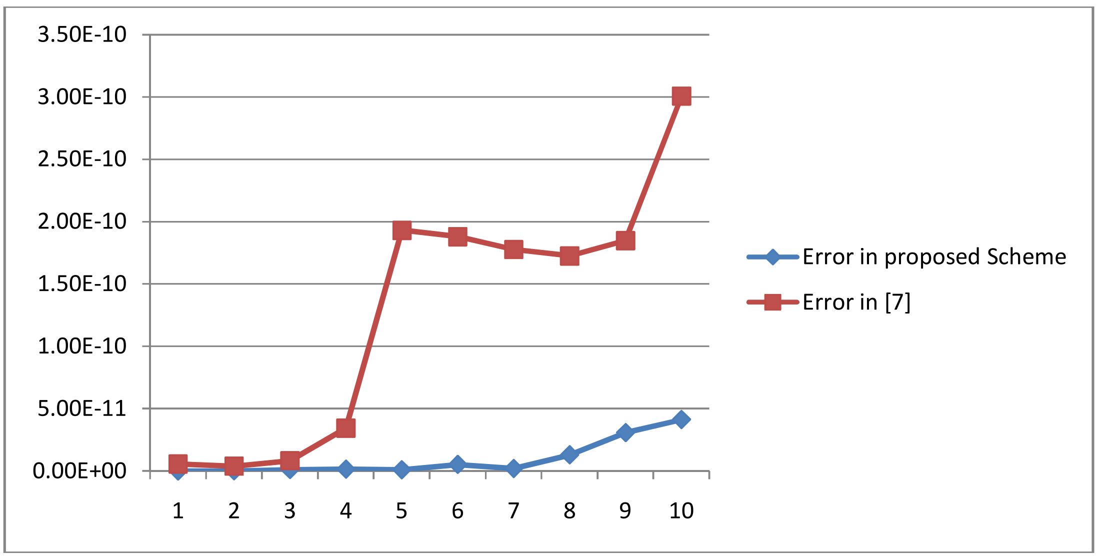

Fig. 2. Plot of error in proposed scheme and error in [7]

7 Discussion of Result

We observed that from the two problems tested with this proposed block hybrid method the results converges to exact solutions and also compared favourably with the existing similar methods (see Tables 1, 2).

8 Conclusion

In this paper, we have presented Hybrid block method algorithm for the solution of first order ordinary differential equations. The approximate solution adopted in this research produced a block method with stability region. This made it to perform well on problems. The block method proposed was found to be zero-stable, consistent and convergent.

Competing Interests

Authors have declared that no competing interests exist.

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