British Journal of Applied Science & Technology
17(3): 1-7, 2016, Article no.BJAST.27524
ISSN: 2231-0843, NLM ID: 101664541

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

On the System of Three Order Rational Difference Equation

Guozhuan Hu[1*]

[1] *Faculty of Mathematics and Physics, Huaiyin Institute of Technology, Huaian, Jiangsu 223003, China.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJAST/2016/27524
Editor(s):
(1) Qing-Wen Wang, Department of Mathematics, Shanghai University, P.R. China.
Reviewers:
(1) Mohamed Mosleh Elysaa Abu Samak, Universiti Kebangsaan Malaysia, Malaysia.
(2) Qianhong Zhang, Guizhou University of Finance and Economics, PR China.
(3) Qamar Din, The University of Poonch Rawalakot, Pakistan.
(4) Jayakumar J, Pondicherry Engineering College, Pondicherry, India.
(5) Iyakino P. Akpan, College of Agriculture, Lafia, Nigeria.
Complete Peer review History: http://www.sciencedomain.org/review-history/15993

Received: 6th June 2016
Accepted: 30th July 2016
Published: 31st August 2016

Original Research Article

Abstract

This paper is concerned with the local and global asymptotic behavior of positive solution for a system of three order rational difference equations

$$x_{n+1} = \frac{x_n}{\alpha + x_{n-1}y_{n-1}}, \quad y_{n+1} = \frac{y_n}{\beta + x_{n-1}y_{n-1}} \quad n = 0, 1, \cdots,$$

where $\alpha, \beta \in (0, \infty)$, and the initial values $x_{-1}, x_0 \in (0, \infty), y_{-1}, y_0 \in (0, \infty)$. Finally, some numerical examples are provided to illustrate theoretical results obtained.

Keywords: Difference equations; equilibrium point; rate of convergence; global asymptotic behavior.

2010 Mathematics Subject Classification: 39A10.

*Corresponding author: E-mail: hugzmath@163.com;

1 INTRODUCTION

Difference equations or discrete dynamical systems are diverse fields which impact almost every branch of pure and applied mathematics. Every dynamical system $x_{n+1} = f(x_n, x_{n-1})$ determines a difference equation and vise versa. Recently, there has been great interest in studying difference equations systems. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations such as population biology [1, 2], economic, probability theory, genetics, psychology, etc. In particular, rational difference equations have appealed more and more scholars due to their wide application. For detail, see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

Kurbanli [3] studied a three-dimensional system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1},\ y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1},$$

$$z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1},$$

where the initial conditions are arbitrary real numbers.

Cinar et al. [4] has obtained the positive solution of the difference equation system

$$x_{n+1} = \frac{m}{y_{\ n}},\ y_{n+1} = \frac{p y_n}{x_{n-1} y_{n-1}}.$$

Cinar [5] has obtained the positive solution of the difference equation system

$$x_{n+1} = \frac{1}{y_{\ n}},\ y_{n+1} = \frac{y_n}{x_{n-1} y_{n-1}}.$$

Also, Cinar [6] has obtained the positive solution of the difference equation system

$$x_{n+1} = \frac{1}{z_n},\ y_{n+1} = \frac{x_n}{x_{n-1}},\ z_{n+1} = \frac{1}{x_{n-1}}.$$

Ozban [7] has investigated the positive solutions of the system of rational difference equations

$$x_{n+1} = \frac{1}{y_{n-k}},\quad y_{n+1} = \frac{y_n}{x_{n-m} y_{n-m+k}}.$$

Papaschinopoulos et al. [8] investigated the global behavior for a system of the following two nonlinear difference equations.

$$x_{n+1} = A + \frac{y_n}{x_{n-p}},$$

$$y_{n+1} = A + \frac{x_n}{y_{n-q}},\ n = 0, 1, \cdots,$$

where A is a positive real number, p, q are positive integers, and $x_{-p}, \cdots, x_0, y_{-q}, \cdots, y_0$ are positive real numbers.

In 2012, Zhang, Yang and Liu [9] investigated the global behavior for a system of the following third order nonlinear difference equations.

$$x_{n+1} = \frac{x_{n-2}}{B + y_{n-2} y_{n-1} y_n},\ y_{n+1} = \frac{y_{n-2}}{A + x_{n-2} x_{n-1} x_n},$$

where $A, B \in (0, \infty)$, and the initial values $x_{-i}, y_{-i} \in (0, \infty), i = 0, 1, 2.$

Although difference equations are sometimes very simple in their forms, they are extremely difficult to understand thoroughly the behavior of their solutions. In book [25] Kocic and Ladas have studied global behavior of nonlinear difference equations of higher order. Similar nonlinear systems of rational difference equations were investigated (see [26]). Other related results reader can refer [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

Motivated by above discussion, our goal, in this paper is to investigate the solutions of the two-dimensional system of three order rational nonlinear difference equations in the form

$$x_{n+1} = \frac{x_n}{\alpha + x_{n-1} y_{n-1}},$$

$$y_{n+1} = \frac{y_n}{\beta + x_{n-1} y_{n-1}},\ n = 0, 1, \cdots. \quad (1.1)$$

where $\alpha, \beta \in (0, \infty)$ and the initial values x_{-1}, x_0, y_{-1} and $y_0 \in (0, \infty)$. Moreover, we have studied the local stability, global stability, boundedness of solutions. We will consider some special cases of (1.1) as applications. Finally, we give some numerical examples.

2 MAIN RESULTS

Let I_x, I_y be some intervals of real number and $f : I_x^2 \times I_y^2 \to I_x, g : I_x^2 \times I_y^2 \to I_y$ be continuously differentiable functions. Then for every initial conditions $(x_i, y_i) \in I_x \times I_y (i = -1, 0)$, the system of difference equations

$$\begin{cases} x_{n+1} = f(x_n, x_{n-1}, y_n, y_{n-1}), \\ y_{n+1} = g(x_n, x_{n-1}, y_n, y_{n-1}), \end{cases} \quad n = 0, 1, 2, \cdots, \tag{2.1}$$

has a unique solution $\{(x_n, y_n)\}_{n=-1}^{\infty}$. A point $(\bar{x}, \bar{y}) \in I_x \times I_y$ is called an equilibrium point of (2.1) if $\bar{x} = f(\bar{x}, \bar{x}, \bar{y}, \bar{y}), \bar{y} = g(\bar{x}, \bar{x}, \bar{y}, \bar{y})$, i. e., $(x_n, y_n) = (\bar{x}, \bar{y})$ for all $n \geq 0$.

Definition 2.1. Assume that $(\bar{x}, \bar{y})$ be a fixed point of (2.1). Then

(i) $(\bar{x}, \bar{y})$ is said to be stable relative to $I_x \times I_y$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that for any initial conditions $(x_i, y_i) \in I_x \times I_y (i = -1, 0)$, with $\sum_{i=-1}^{0} |x_i - \bar{x}| < \delta, \sum_{i=-1}^{0} |y_i - \bar{y}| < \delta$, implies $|x_n - \bar{x}| < \varepsilon, |y_n - \bar{y}| < \varepsilon$.

(ii) $(\bar{x}, \bar{y})$ is called an attractor relative to $I_x \times I_y$ if for all $(x_i, y_i) \in I_x \times I_y (i = -1, 0)$, $\lim_{n\to\infty} x_n = \bar{x}, \lim_{n\to\infty} y_n = \bar{y}$.

(iii) $(\bar{x}, \bar{y})$ is called asymptotically stable relative to $I_x \times I_y$ if it is stable and an attractor.
(iv) Unstable if it is not stable.

Theorem 2.1. *[25] Assume that $X(n + 1) = F(X(n)), n = 0, 1, \cdots$, is a system of difference equations and $\overline{X}$ is the equilibrium point of this system i.e., $F(\overline{X}) = \overline{X}$. If all eigenvalues of the Jacobian matrix J_F, evaluated at $\overline{X}$ lie inside the open unit disk $|\lambda| < 1$, then $\overline{X}$ is locally asymptotically stable. If one of them has modulus greater than one, then $\overline{X}$ is unstable.*

Theorem 2.2. *Assume that $\alpha < 1, \beta < 1$. Then the following statements are true.*
(i) The equilibrium $(0, 0)$ is locally unstable.
(ii) If $\alpha = \beta$, then the system has infinite positive equilibrium points $(\bar{x}, \bar{y})$ such that $\bar{x}\bar{y} = 1 - \alpha$ which are locally unstable.

Proof. (i) We can easily obtain that the linearized system of (1.1) about the equilibrium $(0, 0)$ is

$$\Phi_{n+1} = D\Phi_n, \tag{2.2}$$

where $\Phi_n = (x_n, x_{n-1}, y_n, y_{n-1})^T$,

$$D = (d_{ij})_{4\times4} = \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2.3}$$

The characteristic equation of (2.2) is

$$f(\lambda) = \lambda^2 \left(\lambda - \frac{1}{\alpha}\right)\left(\lambda - \frac{1}{\beta}\right) = 0. \tag{2.4}$$

This shows that the roots of characteristic equation $\lambda = \frac{1}{\alpha}$ and $\lambda = \frac{1}{\beta}$ lie outside unit disk. So the unique equilibrium $(0, 0)$ is locally unstable.
(ii) If $\alpha = \beta$, We can easily obtain that system (1.1) has infinite positive equilibrium points $(\bar{x}, \bar{y})$ such that $\bar{x}\bar{y} = 1 - \alpha$. The linearized system about equilibrium point $(\bar{x}, \bar{y})$ of system (1.1) is

$$\Phi_{n+1} = G\Phi_n, \tag{2.5}$$

where $\Phi_n = (x_n, x_{n-1}, y_n, y_{n-1})^T$,

$$G = \begin{pmatrix} 1 & \alpha - 1 & 0 & -\bar{x}^2 \\ 1 & 0 & 0 & 0 \\ 0 & -\bar{y}^2 & 1 & \beta - 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ denote the 4 eigenvalues of Matrix G. Let $D = \text{diag}(d_1, d_2, d_3, d_4), d_i \neq 0 (i = 1, 2, 3, 4)$ be a diagonal matrix,

Clearly D is invertible. Computing DGD^{-1}, we obtained

$$DGD^{-1} = \begin{pmatrix} 1 & \frac{d_1}{d_2}(\alpha - 1) & 0 & -\frac{d_4}{d_1}\bar{x}^2 \\ \frac{d_2}{d_1} & 0 & 0 & 0 \\ 0 & -\frac{d_3}{d_2}\bar{y}^2 & 1 & \frac{d_3}{d_4}(\beta - 1) \\ 0 & 0 & \frac{d_4}{d_3} & 0 \end{pmatrix}$$

It is well known that G has the same eigenvalues as DGD^{-1}, we obtain that

$$\begin{aligned} \max_{1\leq k\leq 4} |\lambda_k| &= \|DGD^{-1}\| \\ &= \max\left\{ d_2 d_1^{-1}, d_4 d_3^{-1}, 1 + \frac{d_1}{d_2}(1 - \alpha) + \frac{d_4}{d_1}\bar{x}^2, \right. \\ &\quad \left. 1 + \frac{d_3}{d_4}(1 - \beta) + \frac{d_3}{d_2}\bar{y}^2 \right\} \\ &> 1 \end{aligned}$$

It follows from Theorem 2.1 [25] that the positive equilibrium points $(\bar{x}, \bar{y})$ is locally unstable. □

Theorem 2.3. *Assume that $\alpha > 1, \beta > 1$. Then the equilibrium $(0, 0)$ is globally asymptotically stable.*

Proof. For $\alpha > 1, \beta > 1$, from (i) of Theorem 2.2, the equilibrium $(0, 0)$ is locally asymptotically stable. From (1.1), it is easy to see that every positive solution (x_n, y_n) is bounded, i. e., $0 \le x_n \le x_0, 0 \le y_n \le y_0$. Now, it is sufficient to prove that(x_n, y_n) is decreasing. From (1.1), we have

$$\frac{x_{n+1}}{x_n} = \frac{1}{\alpha + x_{n-1}y_{n-1}} \le \frac{1}{\alpha} < 1,$$

$$\frac{y_{n+1}}{y_n} = \frac{1}{\beta + x_{n-1}y_{n-1}} \le \frac{1}{\beta} < 1.$$

This implies that the sequences $\{x_n\}$ and $\{y_n\}$ are decreasing. Hence, $\lim_{n\to\infty} x_n = 0, \lim_{n\to\infty} y_n = 0$. Therefore, the equilibrium $(0, 0)$ is globally asymptotically stable. □

Theorem 2.4. *Assume that $\alpha = \beta = 1$. Then the following statements are true*

(i) the system (1.1) exist infinite equilibrium points $(0, \overline{y})$ and $(\overline{x}, 0)$
(ii) every positive solution (x_n, y_n) of (1.1) converges $(0, 0)$.

Proof. (1) For $\alpha = \beta = 1$, we consider the following system

$$x = \frac{x}{1 + xy}, \quad y = \frac{y}{1 + xy} \tag{2.6}$$

It is clear to see that the system (2.6) has infinite equilibrium points $(0, \overline{y})$ and $(\overline{x}, 0)$. □

(ii) Since the initial values x_0, x_{-1}, y_0, y_{-1} are positive real number. It is similar to the proof of Theorem 2.3. we can easily get the positive solution (x_n, y_n) converges the equilibrium $(0, 0)$.

3 RATE OF CONVERGENCE

In order to study the rate of convergence of positive solutions of (1.1) which converge to equilibrium point $(0, 0)$ of this system, first we consider the following results that gives the rate of convergence of solution of a system of difference equations.

$$X_{n+1} = [A + B(n)]X_n, \tag{3.1}$$

where X_n is m dimensional vector, $A \in C^{m\times m}$ is a constant matrix. $B : Z^+ \to C^{m\times m}$ is a matrix function satisfying

$$\|B(n)\| \to 0, \tag{3.2}$$

as $n \to \infty$, where $\|\cdot\|$ is any matrix norm which is associated with the vector norm

$$\|(x, y)\| = \sqrt{x^2 + y^2}.$$

Proposition 3.1. *(Perrons Theorem)[27] Suppose that condition (3.2) holds. If X_n is any solution of (3.1), then $X_n = 0$ for all large n or*

$$\rho = \lim_{n\to\infty} \frac{\|X_{n+1}\|}{\|X_n\|} \tag{3.3}$$

exists and is equal to the modulus of one of the eigenvalues of matrix A.

Proposition 3.2. *[27] Suppose that condition (3.2) holds. If X_n is any solution of (3.1), then $X_n = 0$ for all large n or*

$$\rho = \lim_{n\to\infty} \sqrt[n]{\|X_{n+1}\|} \tag{3.4}$$

exists and is equal to the modulus of one of the eigenvalues of matrix A.

Let(x_n, y_n) be an arbitrary positive solution of system (1.1) such that $\lim_{n\to\infty} x_n = 0, \lim_{n\to\infty} y_n = 0$. It follows from (1.1) that

$$x_{n+1} - 0 = \frac{x_n}{\alpha + x_{n-1}y_{n-1}} = \frac{1}{\alpha + x_{n-1}y_{n-1}} x_n$$

and

$$y_{n+1} - 0 = \frac{y_n}{\beta + x_{n-1}y_{n-1}} = \frac{1}{\beta + x_{n-1}y_{n-1}} y_n$$

Let $E_n^1 = x_n - 0, E_n^2 = y_n - 0$, then we have

$$E_{n+1}^1 = A_n E_n^1 + B_n E_n^2, \quad E_{n+1}^2 = C_n E_n^1 + D_n E_n^2.$$

where

$$A_n = \frac{1}{\alpha + x_{n-1}y_{n-1}}, B_n = 0, C_n = 0,$$

$$D_n = \frac{1}{\beta + x_{n-1}y_{n-1}}.$$

Moreover

$$\lim_{n\to\infty} A_n = \frac{1}{\alpha}, \quad \lim_{n\to\infty} D_n = \frac{1}{\beta}.$$

Now the limiting system of error terms can be written as

$$\begin{pmatrix} E^1_{n+1} \\ E^2_{n+1} \end{pmatrix} = \begin{pmatrix} 1/\alpha & 0 \\ 0 & 1/\beta \end{pmatrix} \begin{pmatrix} E^1_n \\ E^2_n \end{pmatrix},$$

which is similar to linearized system of (1.1) about the equilibrium point $(0, 0)$.

Using Proposition 3.1 and Proposition 3.2, we have following result.

Theorem 3.1. *Assume that* (x_n, y_n) *be a positive solution of (1.1) such that* $\lim_{n\to\infty} x_n = 0, \lim_{n\to\infty} y_n = 0$*, then the error vector* $E_n = (E^1_n, E^2_n)^T$ *of every solution of (1.1) satisfies the following asymptotic relations*

$$\lim_{n\to\infty} \sqrt[n]{\|E_n\|} = |\lambda_{1,2} F_J(0, 0)|,$$
$$\lim_{n\to\infty} \frac{\|E_{n+1}\|}{\|E_n\|} = |\lambda_{1,2} F_J(0, 0)|,$$

where $\lambda_{1,2} F_J(0, 0) = \frac{1}{\alpha}$ *or* $\frac{1}{\beta}$ *are the characteristic of Jacobian matrix* $F_J(0, 0)$*.*

4 NUMERICAL EXAMPLES

In order to illustrate the results of the previous sections and to support our theoretical discussions, some interesting numerical examples are considered in this section. These examples represent different types of qualitative behavior of solutions to the system of nonlinear difference equations.

Example 4.1. *If the initial conditions* $x_0 = 0.7, x_{-1} = 0.8, y_0 = 0.9, y_{-1} = 0.5$*, and* $\alpha = 1.4, \beta = 1.2$*, we have the following system*

$$x_{n+1} = \frac{x_n}{1.4 + x_{n-1}y_{n-1}}, \; y_{n+1} = \frac{y_n}{1.2 + x_{n-1}y_{n-1}}.$$

It is clear that $\alpha > 1, \beta > 1$*. Then the equilibrium* $(0, 0)$ *is globally asymptotically stable.(Using MATLAB software, See Theorem 2.3 , Fig. 1)*

Example 4.2. *If the initial conditions* $x_0 = 9.8, x_{-1} = 7.2, y_0 = 9.6, y_{-1} = 6.2,$ *and* $\alpha = 0.8, \beta = 0.7$*, we have the following system*

$$x_{n+1} = \frac{x_n}{0.8 + x_{n-1}y_{n-1}}, \; y_{n+1} = \frac{y_n}{0.7 + x_{n-1}y_{n-1}}.$$

It is clear that $\alpha < 1, \beta < 1$*. Then equilibrium* $(0, 0)$ *and* $(\bar{x}, \bar{y})$ *are unstable.(Using MATLAB software, see Theorem 2.2, Fig. 2)*

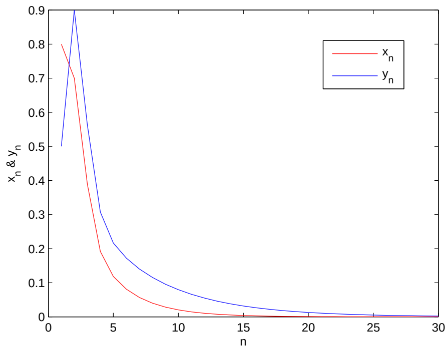

Fig. 1. The fixed point (0,0) is globally asymptotically stable

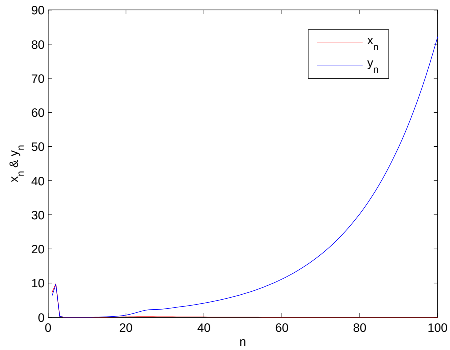

Fig. 2. The fixed point (0,0) and $(\bar{x}, \bar{y})$ is unstable

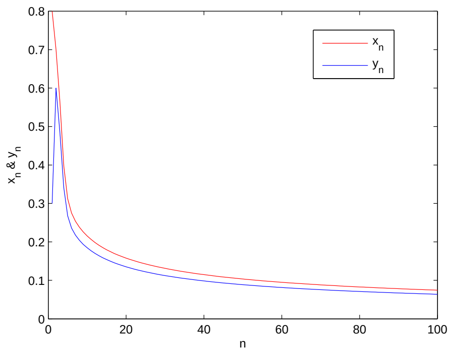

Fig. 3. the positive solution (x_n, y_n) of system (1.1) converges the equilibrium $(0, 0)$.

Example 4.3. *If the initial conditions* $x_0 = 0.7, x_{-1} = 0.8, y_0 = 0.6, y_{-1} = 0.3,$ *and* $\alpha = \beta = 1$*, we have the following system*

$$x_{n+1} = \frac{x_n}{1 + x_{n-1}y_{n-1}}, \; y_{n+1} = \frac{y_n}{1 + x_{n-1}y_{n-1}}.$$

It is clear that $\alpha = \beta = 1$. Then the positive solution (x_n, y_n) of system (1.1) converges the equilibrium $(0, 0)$. (Using MATLAB software, see Theorem 2.4, Fig. 3)

5 CONCLUSION

This paper is concerned with the behavior of positive solution to system (1.1) under some conditions. The results obtained are as follows:

(i) If $\alpha > 1$ and $\beta > 1$, the system (1.1) has an unique equilibrium $(0, 0)$ which is globally asymptotically stable. (ii) If $\alpha < 1$ and $\beta < 1$, then system (1.1) has equilibrium $(0, 0)$ which is unstable. Furthermore if $\alpha = \beta < 1$, then system has infinite positive equilibrium $(\bar{x}, \bar{y})$ such that $\bar{x}\bar{y} = 1 - \alpha$ which are locally unstable. (iii) If $\alpha = \beta = 1$, system (1.1) has infinite equilibrium point $(0, \overline{y})$ and $(\overline{x}, 0)$ and every positive solution (x_n, y_n) converges equilibrium point $(0, 0)$.

ACKNOWLEDGEMENT

The author would like to thank the Editor and the anonymous referees for their careful reading and constructive suggestions.

COMPETING INTERESTS

Author has declared that no competing interests exist.

References

[1] Hassell MP, Comins HN. Discrete time models for two-species competition. Theoretical Population Biology. 1976;9(2):202-221.

[2] Ibrahim TF. Two-dimensional fractional system of nonlinear difference equations in the modeling competitive populations. International Journal of Basic & Applied Sciences. 2012;12(5):103-121.

[3] Kurbanli AS. On the behavior of solutions of the system of rational difference equations: $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1), Y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$ and $z_{n+1} = z_{n-1}/(y_n z_{n-1} - 1)$. Discrete Dynamics in Nature and Society; 2011. Article ID 932632, 12 pages, 2011.

[4] Cinar C, Yalcinkaya I, Karatas R. On the positive solutions of the difference equation system $x_{n+1} = \frac{m}{y_n}, y_{n+1} = \frac{py_n}{x_{n-1}y_{n-1}}$. J. Inst. Math. Comp. Sci. 2005;18:135-136.

[5] Cinar C. On the positive solutions of the difference equation system $x_{n+1} = \frac{1}{y_n}, y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}$. Applied Mathematics and Computation. 2004;158:303-305.

[6] Cinar C, Yalcinkaya I. On the positive solutions of the difference equation system $x_{n+1} = \frac{1}{z_n}, y_{n+1} = \frac{x_n}{x_{n-1}}, z_{n+1} = \frac{1}{x_{n-1}}$. International Mathematical Journal. 2004;5:525-527.

[7] Ozban AY. On the positive solutions of the system of rational difference equations $x_{n+1} = \frac{1}{y_{n-k}}, y_{n+1} = \frac{y_n}{x_{n-m}y_{n-m-k}}$. J. Math. Anal. Appl. 2006;323:26-32.

[8] Papaschinopoulos G, Schinas CJ. On a system of two nonlinear difference equations. J. Math. Anal. Appl. 1998;219:415-426.

[9] Zhang Q, Yang L, Liu J. Dynamics of a system of rational third-order difference equation. Advances in Difference Equations. 2012;136:1-8.

[10] Kurbanli AS, Cinar C, Yalcinkaya I. On the behavior of positive solutions of the system of rational difference equations: $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1), Y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$. Mathematical and Computer Modelling. 2011;53:1261-1267.

[11] Kurbanli AS, Cinar C, Simssek D. On the periodicty of solutions of the system of rational difference equations $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1), Y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$. Applied Mathematics. 2011;2:410-441.

[12] Liu K, Zhao Z, Li X, Li P. More on three-dimensional systems of rational difference equations. Discrete Dynamics in Nature and Society; 2011. Article ID 178483, 9 pages.

[13] Zhang Q, Zhang W. On a system of two high-order nonlinear difference equations. Advances in Mathematical Physics; 2014. Article ID 729273, 8 pages.

[14] Zhang Q, Zhang W, Shao Y, Liu J. On the system of high order rational difference equations. International Scholarly Research Notices; 2014. Article ID 760502, 5 pages.

[15] Zhang Q, Liu J, Luo Z. Dynamical behavior of a system of third-order rational difference equation. Discrete Dynamics in Nature and Society. 2015;2015:1-6.

[16] Ibrahim TF, Zhang Q. Stability of an anti-competitive system of rational difference equations. Archives Des Sciences. 2013;66:44-58.

[17] Zayed KNE, El-Moneam NA. On the global attractivity of two nonlinear difference equations. J. Math. Sci. 2011;177:487-499.

[18] Touafek N, Elsayed KN. On the periodicity of some systems of nonlinear difference equations. Bull. Math. Soc. Sci. Math. Roumanie. 2012;2:217-224.

[19] Touafek N, Elsayed EN. On the solutions of systems of rational difference equations. Mathematical and Computer Modelling. 2012;55:1987-1997.

[20] Kalabusic S, Kulenovic MRS, Pilav K. Dynamics of a two-dimensional system of rational difference equations of Leslie-Gower type. Advances in Difference Equations; 2011.
DOI: 10.1186/1687-1847-2011-29

[21] Din Q. Global behavior of a plant-herbivore model. Advances sin Difference Equations. 2015;1:1-12.

[22] Din Q, Khan KA, Nosheen A. Stability analysis of a system of exponential difference equations. Discrete Dyn. Nat. Soc. Volume 2014, Article ID 375890, 11 pages.

[23] Din Q. Global stability of a population model. Chaos Soliton Fract. 2014;59:119-128.

[24] Din Q, Donchev T. Global character of a host-parasite model. Chaos Soliton Fract. 2013;54:1-7.

[25] Kocic VL, Ladas G. Global behavior of nonlinear difference equations of higher order with application. Kluwer Academic Publishers, Dordrecht; 1993.

[26] Kulenovic MRS, Merino O. Discrete dynamical systems and difference equations with mathematica. Chapman and Hall/CRC, Boca Raton, London; 2002.

[27] Pituk M. More on poincares and Perrons theorems for difference equations. J. Diff. Eq. App. 2002;6:201-216.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/15993