Asian Research Journal of Mathematics

10(4): 1-9, 2018; Article no.ARJOM.43203
ISSN: 2456-477X

Application of JWKB Method on the Effect of Magnetic Field on Alpha Decay

A. T. Ngiangia[1], M. A. Orukari[2*] and F. Jim-George[3]

[1]University of Port Harcourt, P.M.B. 5323, Choba, Port Harcourt, Nigeria.
[2]Department of Mathematics and Computer Science, Niger Delta University, Amassoma, Bayelsa State, Nigeria.
[3]Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt, Nigeria.

Authors' contributions

This work was carried out in collaboration between all authors. Author ATN designed the study, performed the mathematical analysis, wrote the protocol and wrote the first draft. Author MAO managed the analysis and authors MAO and FJG managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2018/43203
Editor(s):
(1) Dr. Sheng Zhang, School of Mathematics and Physics, Bohai University, Jinzhou, China.
(2) Dr. Junjie Chen, Department of Electrical Engineering, University of Texas at Arlington, USA.
Reviewers:
(1) Zahir Shah, Ghandhara University of Science and Technology, Pakistan.
(2) Wensheng Zhang, AMSS, Chinese Academy of Sciences, China.
(3) T. O. Adeeko, University of Abuja, Nigeria.
Complete Peer review History: http://www.sciencedomain.org/review-history/26232

Review Article

Received: 02 July 2018
Accepted: 07 September 2018
Published: 14 September 2018

Abstract

The effect of magnetic field on radioactive alpha decay was examined theoretically. The alpha decay and magnetic field potential are added to the time independent Schrodinger wave equation and the Jeffreys, Wentzel, Kramers and Brillouin (JWKB) method was used to examine the radial part of the Schrodinger equation. The determination of the mean lifetime of the alpha decay and the ratio of the radii showed that the magnetic field, influenced the alpha decay using two radioactive elements. The eigenvalues and eigenfunctions of the model also laid credence to the alteration of the reaction.

Keywords: JWKB method; magnetic field; alpha decay; mean lifetime; tunneling probability; radioactivity.

**Corresponding author: E-mail: orukarimer@gmail.com;*

1 Introduction

Magnetic field interactions are gaining more popularity in almost all areas of science and its applications. According to Branover [1], magnetic field exists everywhere in the universe, therefore magneto hydrodynamics phenomena must occur wherever conducting fluids are present. Such interactions occur both in nature and in anthropogenic devices. Examples are, in the sun, in the earth's interior, in the ionosphere, in the propulsion units and power generators [2] to mention few. Spin chemistry also described the magnetic compass of birds and other animals, which enable them to use the Earth's magnetic field as a guide during migration. As a result of extensive study mainly pioneered by Rutherford and Soddy around early nineteenth century, major break through was made about radioactive transformations wherein atoms of elements exhibit alpha decay by the relation ${}^{z}_{a}A \rightarrow {}^{z-4}_{a-2}B + {}^{4}_{2}He$. Where A and B represent elements and He is helium. This result and that of beta decay, as well as gamma decay, are applicable in medical and industrial uses. The importance of magnetic field interactions with particles cannot be overemphasized because its study influences several areas of study. The Zeeman and Paschen-Back effects seen in spectroscopy as a result of the splitting of lines in a magnetic field is one of such. The calculation of diamagnetic and paramagnetic susceptibilities, the experiments on electron spin resonance and nuclear magnetic resonance all have great practical value as seen in solid state physics [3-6]. The kinetic energy of the emitted alpha particle is also obtained accurately by measuring the radius of the path described by the particle in a magnetic field [7]. Rodgers [8] stated that chemical reactions that involve radical intermediates are influenced by magnetic fields, which act to alter their rate, yield or product distribution. Steiner and Ulrich [9] and Hayashi [10], also opined that the magnetic field also alter the rate and yield of chemical reactions. Filip [11], carried out a study and said, magnetic field effects have been a successful tool for studying carrier dynamics in organic semiconductors as the weak spin-orbit coupling is strong in organic-inorganic hybrid perovskites, which is a promising material for photovoltaic and light-emitting applications. Several studies are abounded in Ghatak and Lokanathan [12], where JWKB method was used to analyze alpha decay and the energy levels and wave functions of two dimensional harmonic oscillators placed in a uniform magnetic field was discussed. In considering the three types of radiations, the distinguishing characters of penetrating power, deflections in electric or magnetic field and determination of specific charge, it is observed that the alpha particles are deflected by magnetic field. Based on the premise that magnetic field affects the deflection of alpha particle and other studies cited, it is reasoned that its decay can also be affected by magnetic field, hence the aim of the study is to examine the effect of magnetic field on alpha decay analytically and compare with experimental results.

2 Experimental Methods

In this section, we employed the Jeffreys, Wentzel, Kramers and Brillouin method often referred to as the JWKB method. The method was successfully used in tackling potentials which are slowly varying [13-18]. According to the JWKB method, the time independent Schrodinger equation of the form:

$$\frac{d^2\psi(r)}{dx^2} + k^2(r)\psi(r) = 0 \tag{1}$$

where $k^2(r) = \frac{2\mu}{\hbar^2}\left[E - V(r)\right]$ (2)

is applicable to potentials which are slowly varying such that

$$\left|\frac{1}{k(r)}\frac{dk}{dr}\right| \le k(r) \tag{3}$$

and the eigenvalues and eigenfunctions are respectively given abounded in Ghatak and Lokanathan [12], Zettili [19] as

$$\int_{b_3}^{b_4} k(r)dr = \left(n + \frac{1}{2}\right)\pi \tag{4}$$

$$\psi(r) = \frac{cons\tan t}{\sqrt{k(r)}} \exp \pm i \int^{r} k(r)dr \tag{5}$$

where ψ is wave function, μ is mass of particle, E is total energy, V(r) is potential, $\hbar$ is planck's constant, n is quantum number.

3 Radial Part of Schrodinger Wave Equation with Potentials

According to Ghatak and Lokanathan [12], E. Schrodinger in his doctoral thesis, transformed the wave equation

$$\nabla^2 \psi = \upsilon^{-2} \frac{\partial^2 \psi}{\partial t^2} \tag{6}$$

in conjunction with De Broglie's waves, into the time-dependent equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2\mu} \nabla^2 \psi + V\psi \tag{7}$$

where υ is wave velocity.

The time-dependent equation of (7) is equation (1) and the spherical coordinate transformation of equation (1) together with the use of separating the variables, transformed it into radial and angular part. Our study is dependent only on the radial part. Therefore, following Ghatak and Lokanathan [12], and including the magnetic field, the radial part of our equation takes the form

$$\frac{d^2 u(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - \frac{zZq^2}{4\pi\varepsilon_0 r} + \frac{qBm\hbar}{2\mu} - \frac{l(l+1)}{2\mu r^2}\right]u(r) = 0 \tag{8}$$

the alpha potential is given as $v(r) = \dfrac{zZq^2}{4\pi\varepsilon_0 r}$ r > R

$= -V_0$ r < R

where R is the radius of the nucleus, zq and Zq are the charges of the alpha particle and the daughter cells respectively, B is the magnetic induction, m is magnetic quantum number, l is quantum number and μ is mass of the alpha particle.

We assume $l = 0$ and the tunneling probability is given as

$$T = \exp(-2\beta) \tag{9}$$

$$\beta = \sqrt{\frac{2\mu}{\hbar^2}}\int_R^a \sqrt{\frac{zZq^2}{4\pi\varepsilon_0 r} - \frac{qBm\hbar}{2\mu} - E dr} \tag{10}$$

Equation (10) can be simplified to take the form

$$\beta = \left(\frac{4\mu^2 zZq^2}{4\pi\varepsilon_0 \left(2\mu E + qBm\hbar\right)\hbar^2}\right)^{0.5} \int_R^a \sqrt{\frac{1}{r} - \frac{1}{a}} dr \tag{11}$$

where $a = \dfrac{2\mu zZq^2}{4\pi\varepsilon_0\left(2\mu E + qBm\hbar\right)}$ (12)

integration of equation (11), yield the expression

$$\beta = \left(\frac{4\mu^2 zZq^2}{4\pi\varepsilon_0 \left(2\mu E + qBm\hbar\right)\hbar^2} a\right)^{0.5} \left[Cos^{-1}\sqrt{\frac{R}{a}} - \sqrt{\frac{R}{a} - \frac{R^2}{a^2}}\right] \tag{13}$$

According to Ghatak and Lokanathan [12], $\frac{R}{a} \le 1$ we can then write the expression inside the square bracket of equation (13) as

$$\left[Cos^{-1}\sqrt{\frac{R}{a}} - \sqrt{\frac{R}{a} - \frac{R^2}{a^2}}\right] \approx \frac{\pi}{2} - \sqrt{\frac{R}{a}} - \sqrt{\frac{R}{a}} \approx \frac{\pi}{2} - 2\sqrt{\frac{R}{a}} \tag{14}$$

The inverse mean lifetime of an alpha particle inside the nucleus is given [12] as

$$\frac{1}{\tau} = \frac{V}{R}\exp(-2\beta) \tag{15}$$

Using equations (12) and (14), equation (13) takes the form

$$\beta = \left(\frac{4\mu^2 zZq^2}{4\pi\varepsilon_0 \left(2\mu E + qBm\hbar\right)\hbar^2} . \frac{2\mu zZq^2}{4\pi\varepsilon_0\left(2\mu E + qBm\hbar\right)}\right)^{0.5} .(\frac{\pi}{2} - \left(\frac{4\pi\varepsilon_0\left(2\mu E + qBm\hbar\right)R}{2\mu zZq^2}\right)^{0.5}) \tag{16}$$

Using the following Ghatak and Lokanathan [12], Zettili [19] constants

$\mu = 1.672631x10^{-27} kg = 938.27231 MeV$

$\hbar = 1{,}05457266x10^{-34} J.s$

$q = 1.60217733x10^{-19} C$

$\varepsilon_0 = 8.8541878x10^{-12} F/m$

$\dfrac{q^2}{4\pi\varepsilon_0\hbar C} = 7.29735308x10^{-3}$

$z = 2$

$m = 1$

$B = 1T$

$C = 3x10^8 m/s$

$\hbar C = 197.327 MeV$

equations (12) and (16) take the form

$$a = \frac{5.4404x10^{-12} Z}{1876E + 5.3405911x10^{-26}} \tag{17}$$

$$\beta = \left(\frac{1.02061904x10^{-8} Z}{(2.938E + 5.3405911x10^{-26})1.05457266x10^{-34}} \cdot \frac{5.4404x10^{-12} Z}{1876E + 5.3405911x10^{-26}} \right)^{0.5} \cdot \left(1.5707963268 - \left(\frac{(1876E + 5.3405911x10^{-26})R}{5.4404x10^{-12} Z} \right)^{0.5} \right) \tag{18}$$

The radius of the nucleus is given Ghatak and Lokanathan [12] as

$$R = (1.07x10^{-13}) A^{\frac{1}{3}} m \tag{19}$$

where A is the mass number of the element

The velocity V can be written as $C\left(\frac{2E}{\mu C^2}\right)^{0.5}$, therefore, equation (15) can be rewritten after taking the natural logarithm on both sides as

$$-Log\tau = Log 6.5420560748 + 0.5 LogE - \frac{1}{3} LogA$$

$$-2\left(\frac{5.5525758252x10^{-20} Z^2}{5.8124754657x10^{-31} E^2 + 1.0582256533x10^{-56} E + 3.0078429974x10^{-85}} \right)^{0.5} \cdot \left(1.5707963268 - \left(\frac{(36.89655172E + 1.0503699134x10^{-27}) A^{\frac{1}{3}}}{Z} \right)^{0.5} \right) \tag{20}$$

According to Gasiorowicz [20], Hyde [21], the most important factor that agrees with experimental results is the exponential term in equation (20), we therefore express it as

$$Log\frac{1}{\tau} = -2\left(\frac{5.5525758252x10^{-20} Z^2}{5.8124754657x10^{-31} E^2 + 1.0582256533x10^{-56} E + 3.0078429974x10^{-85})} \right)^{0.5} \cdot \left(1.5707963268 - \left(\frac{(36.89655172E + 1.0503699134x10^{-27}) A^{\frac{1}{3}}}{Z} \right)^{0.5} \right) \tag{21}$$

Illustration with radioactive decay equation Ghatak and Lokanathan [12] of Polonium as

$$ {}^{212}_{84}Po \rightarrow {}^{208}_{82}Pb + {}^{4}_{2}He(8.9MeV) \tag{22}$$

Substituting for A, Z and E in equation (20), the mean lifetime of the alpha decay is given by

$$\tau = \exp(-16434880)s \tag{23}$$

The corresponding experimental value Ghatak and Lokanathan [12] is $\tau = 3x10^{-7}s$

$$\frac{R}{a} = 19.8601 \tag{24}$$

A second illustration of Uranium decay Ghatak and Lokanathan [12] is given by

$$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He(4.2MeV) \tag{25}$$

Again, if the values of A, Z and E is put into equation (20), the result will be

$$\tau = \exp(-18590460)s \tag{26}$$

The experimental value Ghatak and Lokanathan [12] is $\tau = 10^{17}s$

$$\frac{R}{a} = 8.84621 \tag{27}$$

4 Energy Eigenvalues and Eigenvectors

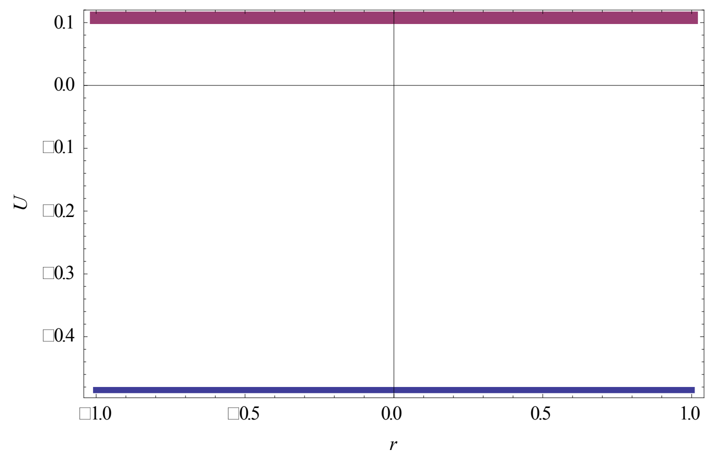
Considering equation (16), quantization of the energy levels of bound states for potentials with no rigid walls as the case of our present study, using the constants [22-24] $\mu = \hbar = 1$, z = 2, m = 1, and B = 1, applying equation (17) and putting the relation in equation (4), the result is

$$2E = \frac{-1.712x10^{-12}A^{\frac{1}{3}} \pm \sqrt{2.931x10^{-24}A^{\frac{2}{3}} - 64\pi^2 Zq^2\left[\left(n+\frac{1}{2}\right)\pi\right]^2}}{4\pi\varepsilon_0\left[\left(n+\frac{1}{2}\right)\pi\right]^2} - q \tag{28}$$

and

$$U(r) = \frac{C}{\left(2E - \frac{4.613x10^{-28}Z}{r} + 8.01x10^{-20}\right)^{0.25}} Sin[\beta] \tag{29}$$

where C is constant not normalized.



KEY: Dark red line ......A =208,Z= 82.Blue line.....A = 234, Z = 90

Fig. 1. The dependence of radial part on r with A = 208, Z = 82, E = 8.9 to A = 234, Z = 90, E = 4.2 and C = 1

5 Discussion

In a region where the magnetic field strength is about $B \geq 1$, the decay of alpha particle is greatly hindered. The ratio $\frac{R}{a}. >>>> 1$ also laid credence to the alpha decay as demonstrated with the two radioactive elements. The study also reveals that, the higher, the mass number of the decayed element, the higher the departure from mean life time from experimental results. The results are also related to the observation of the study of Wan et al. [25], in which it was reported that, strong magnetic field and other factors changed the decay energy and daughter nucleus in external environments.

6 Conclusion

The model showed the effect of magnetic field on alpha decay theoretically. Several studies in the literature reported that magnetic field alter the rate of chemical reaction, our study is also a chemical reaction but tackled analytically using magnetic field, the yield of alpha decay can be controlled. The study sheds light on the use of models in verifying experimental results. The $l \neq 0$ case should also be considered to give a holistic view of the mean lifetime of any alpha decay element.

Acknowledgement

The authors thanked all reviewers for the useful comments.

Competing Interests

Authors have declared that no competing interests exist.

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