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Using $C-$Class Function on Coupled Fixed Point Theorems for Mixed Monotone Mappings in Partially Ordered Rectangular Quasi Metric Spaces

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Authors' contributions

This work was carried out in collaboration between all authors. Author AM is a supervisor and authors BM and SA are students of him. Authors AM, BM and SA wrote the first draft of the manuscript. Authors AM and BM managed and edited the manuscript. All authors read and approved the final manuscript.

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Original Research Article

Abstract

In the manuscript, using $C-$class function, a concept of a mixed monotone mapping is acquainted and a coupled fixed point theorems is substantiated for such nonlinear shrinkage mappings in partially ordered exact rectangular metric spaces. We enlarge and universalize the conclusions of [9].

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1 Introduction

Stefan Banach is a Polish mathematician, who described a benchmark fixed point theorem familiar with the Banach Shrinkage Principle (BSP). The BSP has been universalized in many different subjects. Many researcher enlarged to the situation of nonlinear shrinkage mappings. The presence of a fixed point studied in many metric spaces and imagined in many research paper and applied to many subjects that is, mathematics, engineering and economy.

In [1], some fixed point theorems were proved on complete and compact metric spaces and also given some examples. Fixed points of a generalised weakly contractive map was established the existence in $T-$ orbitally in completed metric spaces with respect to boundary spaces.

Some existence theorems of the couple fixed point were presented for both continuous and discontinuous operators so some application to the initial value problems of ordinary differential equations with discontinuous right-hand sides were investigated in [2].

In [3], using a weak contractivity type of supposition, a fixed point theorem is verified for mixed monotone mapping in a metric space endowed with partial order. For a periodic boundary value problem, the existence and uniqueness of solution is studied.

The f is continuous function of $C-$ class, which is described in [4], and also some theorems, consequences and examples are given. A common fixed point theorem was testified for multi-valued and single- valued mappings in a dislocated metric spaces verifying a weak contractive condition with function of C with respect to $F-\varphi-$weak contractively condition in [5].

A investigation of fixed point results was given in generalized metric spaces with respect to a fixed point theorem of Banach-Caccipoli type on a class of generalized metric spaces without Hausdorff in [6].

In [7], some couple fixed point results are verified under $c-$ distance satisfying certain contractive condition in cone metric space and their results are generalized the conclusion fixed point theorems of single value mapping for $c-$ distance in cone metric space and also an example is given there.

Some fixed point theorems were introduced for cyclic admissible generalized contractions involving $C-$class functions and admissible mappings in metric-like spaces. The results which extended and improved many recent results in the literature. They also presented some examples and applications to functional equations arising in dynamic programming in [8].

The presence of a coupled fixed point theorem is proved for a mixed monotone mapping $T : X\times X \to X$ under a universalized contraction and build the exclusiveness under a supplementary assumption on partially ordered complete rectangular metric space in [9].

In this article using $C-$class function, we prove a coupled fixed point theorem for mixed monotone mapping $F-\psi-$of $C-$ class function in partial ordered rectangular quasi metric space. The proffered theorem extend and unify various known fixed point conclusion.

2 Mathematical Preliminaries

We state primary definitions and notations to be used throughout the article, where N is the non-negative set of integers. Rectangular quasi metric spaces are the following designated.

Definition 2.1. Assume X is not a null set and $d : X \times X \to [0, \infty]$ satisfy the following conditions for all $x, y \in X$ and all distinct $u, v \in X$ each of which is dissimilar from x and y.

($\mathrm{R_qM1}$) $d(x, y) = 0 \Leftrightarrow x = y$,

($\mathrm{R_qM2}$) $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$.

At that time the map d is named a rectangular quasi metric on X ($rq-$metric) and the pair (X, d) is named a rectangular quasi metric spaces (R_qMS).

Definition 2.2. A sequence $\{x_n\}$ in $rq-$metric space (rectangular quasi-metric space) (X, d) is named Cauchy if given $\varepsilon > 0, n_0 \in N$ with for all $n, m \geq n_0$, implies $d(x_n, x_m) < \varepsilon$ or $d(x_m, x_n) < \varepsilon$, namely $\min\{d(x_n, x_m), d(x_m, x_n)\} < \varepsilon$ in [9].

In this instance x is named the rq limit of $\{x_n\}$.

Definition 2.3. A $rq-$metric space space (X, d) is named complete if every Cauchy sequence in it is rq convergent in [9].

Lemma 2.1. *Assume (X, d) is a $rq-$metric space. If $f : X \to X$ is a contraction map, at the time $\{f^n(x_0)\}$ is a Cauchy sequence for each $x_0 \in X$ in [9].*

Definition 2.4. Assume $(X, \preceq)$ is a partially ordered set and (X, d) Hausdorff and completed R_qMS. Let $T : X \times X \to X$. If T possesses the mixed monotone property, then for any $x, y \in X, T(x, y)$ is monotone non-decreasing with respect to x, and also it is monotone nonincreasing with respect to y for all $x, y \in X$ such that

$$x_1, x_2, y \in X, x_1 \preceq x_2 \Longrightarrow T(x_1, y) \preceq T(x_2, y)$$

and

$$y_1, y_2, x \in X, y_1 \preceq y_2 \Longrightarrow T(x, y_1) \succeq T(x, y_2)$$

in [9].

Definition 2.5. If $T(x, y) = x$ and $T(y, x) = y$ then a member $(x, y) \in X \times X$ is named a coupled fixed point of the mapping T in [9].

For any $(x, y), (u, v) \in X \times X$, the multiply space $X \times X$ is equipped with the metric ρ described by

$$\rho((x, y), (u, v)) = \frac{d(x, u) + d(y, v)}{2}$$

So ρ is a $rq-$metric.

Theorem 2.2. *Assume (X, d) is a complete $rq-$metric space and $f : X \to X$ is a continuous contraction map. In this case f possesses an individual fixed point in [9].*

The concept of $C-$class functions (see Definition 2.6) was introduced by H. Ansari in [4] and is important, for example (1),(2),(9) and (15) from Example 2.3. Also see [5], [10] and [8].

Definition 2.6. [4] A continuous function $F : [0,\infty)^2 \to \mathbb{R}$ is called $C-$function if for any $s,t \in [0,\infty)$, the following conditions hold:
(1) $F(s,t) \leq s$;
(2) $F(s,t) = s$ implies that either $s = 0$ or $t = 0$.

An extra condition on F that $F(0,0) = 0$ could be imposed in some cases if required. The letter $\mathcal{C}$ will denote the class of all $C-$ functions.

Example 2.3. *[4] Following examples show that the class $\mathcal{C}$ is nonempty:*

1. $F(s,t) = s - t$.
2. $F(s,t) = ms, 0 < m < 1$
3. $F(s,t) = \frac{s}{(1+t)^r}$ for some $r \in (0,\infty)$.
4. $F(s,t) = \log(t + a^s)/(1 + t)$, for some $a > 1$.
5. $F(s,t) = \ln(1 + a^s)/2$, for $a > e$. Indeed $F(s,1) = s$ implies that $s = 0$.
6. $F(s,t) = (s + l)^{(1/(1+t)^r)} - l$, $l > 1$, for $r \in (0,\infty)$.
7. $F(s,t) = s\log_{t+a} a$, for $a > 1$.
8. $F(s,t) = s - (\frac{1+s}{2+s})(\frac{t}{1+t})$.
9. $F(s,t) = s\beta(s)$, where $\beta : [0,\infty) \to [0,1)$.
10. $F(s,t) = s - \frac{t}{k+t}$.
11. $F(s,t) = s - \varphi(s)$, where $\varphi : [0,\infty) \to [0,\infty)$ is a continuous function such that $\varphi(t) = 0$ if and only if $t = 0$.
12. $F(s,t) = sh(s,t)$, where $h : [0,\infty) \times [0,\infty) \to [0,\infty)$ is a continuous function such that $h(t,s) < 1$ for all $t,s > 0$.
13. $F(s,t) = s - (\frac{2+t}{1+t})t$.
14. $F(s,t) = \sqrt[n]{\ln(1 + s^n)}$.
15. $F(s,t) = \phi(s)$, where $\phi : [0,\infty) \to [0,\infty)$ is a upper semicontinuous function such that $\phi(0) = 0$ and $\phi(t) < t$ for $t > 0$.
16. $F(s,t) = \frac{s}{(1+s)^r}$; $r \in (0,\infty)$.

Definition 2.7. A function $\psi : [0,\infty) \to [0,\infty)$ is called an altering distance function if the following properties are satisfied:
(i) ψ is non-decreasing and continuous,
(ii) $\psi(t) = 0$ if and only if $t = 0$ in [1].

Remark 2.1. We denote altering distance function as Ψ.

Definition 2.8. An φ ultra altering distance function is a continuous, nondecreasing mapping $\varphi : [0,\infty) \to [0,\infty)$ such that $\varphi(t) > 0$, $t > 0$ and $\varphi(0) \geq 0$ in [4].

Remark 2.2. We denote ultra altering distance function as Φ_u.

Lemma 2.4. *Suppose (X,d) is a rectangular metric space. Let $\{x_n\}$ be a sequence in X such that $d(x_n,x_{n+1}) \to 0$ as $n \to \infty$. If $\{x_n\}$ is not a Cauchy sequence then there exist an $\varepsilon > 0$ and sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ with*

$m(k) > n(k) > k$ such that $d(x_{m(k)}, x_{n(k)}) \geq \varepsilon$, $d(x_{m(k)-1}, x_{n(k)}) < \varepsilon$ and
(i) $\lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)+1}) = \varepsilon$;
(ii) $\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = \varepsilon$;
(iii) $\lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)}) = \varepsilon$
in [[9]].

Remark 2.3. We note that also can see $\lim_{k\to\infty} d(x_{m(k)+1}, x_{n(k)+1}) = \varepsilon$ and
$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)-1}) = \varepsilon$.

3 Main Consequences

In next section, we give a universalization in Theorem 2.1 of [9] for a mixed monotone mappings. We verify a coupled fixed point theorem in exact partially ordered rectangular quasi metric spaces. Now the main conclusion is certified.

Theorem 3.1. *Supposing $(X, \preceq)$ is partially ordered set and (X, d) Hausdorff and exact partially ordered rectangular quasi metric spaces. Suppose there subsist a function $\psi, \phi : [0, \infty) \to [0, \infty)$ with $\psi(t) < t$, $\phi(t) < t$ and for each $t > 0$ $\lim_{r\to t^+} \psi(r) < t$, $\lim_{r\to t^+} \phi(r) < t$ and also the mixed monotone property on X is verified by $T : X \times X \to X$ be a continuous mapping and*

$$d(T(x, y), T(u, v)) \leq F(\psi(\rho((x, y), (u, v))), \phi(\rho((x, y), (u, v)))) \tag{3.1}$$

for all $x, y, u, v \in X$ for which $x \preceq u$, $y \succeq v$ There subsist $(x_0, y_0) \in X \times X$ with $x_0 \prec T(x_0, y_0)$, $y_0 \succeq T(y_0, x_0)$. So T has got an individual coupled fixed point.

Proof. Suppose $x_0, y_0 \in X$ is with $x_0 \prec T(x_0, y_0)$, $y_0 \succeq T(y_0, x_0)$. Let $x_1 = T(x_0, y_0)$, $y_1 = T(y_0, x_0)$. Then $x_0 \prec x_1$, $y_0 \succeq y_1$. Again, let $x_2 = T(x_1, y_1)$, $y_2 = T(y_1, x_1)$. The mixed monotone property is verified by T, obtaining $x_1 \prec x_2$, $y_1 \succeq y_2$. To continue to do so through, two sequences $\{x_n\}$, $\{y_n\}$ are built in X with $x_{n+1} = T(x_n, y_n)$, $y_{n+1} = T(y_n, x_n)$,

$$x_0 \prec x_1 \prec x_2 \prec \cdots \prec x_n \prec x_{n+1} \prec \cdots \tag{3.2}$$

and,

$$y_0 \succeq y_1 \succeq y_2 \succeq \cdots \succeq y_n \succeq y_{n+1} \succeq \cdots . \tag{3.3}$$

Denote

$$\delta_n = \rho((x_n, y_n), (x_{n+1}, y_{n+1})) = \frac{d(x_n, x_{n+1}) + d(y_n, y_{n+1})}{2}. \tag{3.4}$$

We expose that $\delta_n < \delta_{n-1}$. Now, enforcing if the inequality (3.1) is implemented with $(x, y) = (x_n, y_n)$, $(u, v) = (x_{n-1}, y_{n-1})$, for all $n \geq 0$. Utilizing properties of ψ, we get

$$\begin{aligned} d(x_{n+1}, x_n) &= d(T(x_n, y_n), T(x_{n-1}, y_{n-1})) \\ &\leq F(\psi(\rho((x_n, y_n), (x_{n-1}, y_{n-1}))), \phi(\rho((x_n, y_n), (x_{n+1}, y_{n+1})))) \\ &\leq \psi(\rho((x_n, y_n), (x_{n-1}, y_{n-1}))) \\ &< \rho((x_n, y_n), (x_{n-1}, y_{n-1})). \end{aligned} \tag{3.5}$$

Similarly, we can obtain

$$d(y_{n+1}, y_n) < \rho((x_n, y_n), (x_{n-1}, y_{n-1})). \tag{3.6}$$

Thus we acquire

$$\frac{d(x_{n+1}, x_n) + d(y_{n+1}, y_n)}{2} < \rho((x_n, y_n), (x_{n-1}, y_{n-1}).$$

That is $\delta_n < \delta_{n-1}$. Thus $\{\delta_n\}$ is monotone decreasing bounded to the bottom. Hereby, there subsist a $\delta \geq 0$ with

$$\lim_{n\to\infty} \delta_n = \delta.$$

Demonstrating $\delta = 0$. Supposing the contrary $\delta > 0$. At the time from (3.1) obtaining

$$\begin{aligned} d(x_{n+1}, x_n) &= d(T(x_n, y_n), T(x_{n-1}, y_{n-1})) \\ &\leq F(\psi(\rho((x_n, y_n), (x_{n-1}, y_{n-1}))), \phi(\rho((x_n, y_n), (x_{n+1}, y_{n+1})))) \\ &\leq \psi(\rho((x_n, y_n), (x_{n-1}, y_{n-1}))). \end{aligned} \tag{3.7}$$

Similarly, we can obtain

$$d(y_{n+1}, y_n) < \psi(\rho((x_n, y_n), (x_{n-1}, y_{n-1}))). \tag{3.8}$$

Thus we get

$$\frac{d(x_{n+1}, x_n) + d(y_{n+1}, y_n)}{2} < \rho((x_n, y_n), (x_{n-1}, y_{n-1}). \tag{3.9}$$

While $n \to \infty$ in (3.9), getting

$$\delta = \lim_{n\to\infty} \delta_n < \lim_{n\to\infty} \delta_{n-1} = \delta.$$

So the incompatibility is obtained. Hence $\delta = 0$. Namely

$$\lim_{n\to\infty} \rho((x_n, y_n), (x_{n+1}, y_{n+1})) = \lim_{n\to\infty} \frac{d(x_{n+1}, x_n) + d(y_{n+1}, y_n)}{2} = 0.$$

Thus

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0 \text{ and } \lim_{n\to\infty} d(y_n, y_{n+1}) = 0. \tag{3.10}$$

Evidencing $\{x_n\}$, $\{y_n\}$ are partially ordered rectangular quasi metric spaces Cauchy sequences. To accept the opposite that at least one of $\{x_n\}$ or $\{y_n\}$ is not a partially ordered rectangular quasi metric spaces Cauchy sequences. At the time there subsist an $\epsilon > 0$ when obtaining two subsequences $\{(x_{n(i)}\}$ and $\{(x_{m(i)}\}$ of $\{x_n\}$ with $n(i)$ is the smallest index where

$$n(i) > m(i) > i, \quad d(x_{m(i)}, x_{n(i)}) + d(y_{m(i)}, y_{n(i)}) \geq \epsilon. \tag{3.11}$$

This means that

$$d(x_{m(i)}, x_{n(i)-1}) + d(y_{m(i)}, y_{n(i)-1}) < \epsilon. \tag{3.12}$$

By (rectangular quasi metric space 2), we obtain

$$\begin{aligned} d(x_{m(i)}, x_{n(i)}) &\leq d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)-1}, x_{n(i)-1}) \\ &\quad + d(x_{n(i)-1}, x_{n(i)}). \end{aligned} \tag{3.13}$$

Similarly from (rectangular quasi metric space 2), we can obtain

$$\begin{aligned} d(y_{m(i)}, y_{n(i)}) &\leq d(y_{m(i)}, y_{m(i)-1}) + d(y_{m(i)-1}, y_{n(i)-1}) \\ &\quad + d(y_{n(i)-1}, y_{n(i)}). \end{aligned} \tag{3.14}$$

By adding (3.13) and (3.14), from (3.10), (3.11) and (3.12)

$$\begin{aligned} d(x_{m(i)}, x_{n(i)}) + d(y_{m(i)}, y_{n(i)}) \leq\ & [d(x_{m(i)}, x_{m(i)-1}) + d(y_{m(i)}, y_{m(i)-1})] \\ & +[d(x_{m(i)-1}, x_{n(i)-1}) + d(y_{m(i)-1}, y_{n(i)-1})] \\ & +[d(x_{n(i)-1}, x_{n(i)}) \\ & +d(y_{n(i)-1}, y_{n(i)})]. \end{aligned} \tag{3.15}$$

Getting the limit as $i \to \infty$ in (3.15), obtaining by (3.10), (3.11)

$$\begin{aligned} \epsilon \leq \lim_{i\to\infty}[d(x_{m(i)}, x_{n(i)}) + d(y_{m(i)}, y_{n(i)})] \leq\ & \lim_{i\to\infty}[d(x_{m(i)-1}, x_{n(i)-1}) \\ & +d(y_{m(i)-1}, y_{n(i)-1})]. \end{aligned} \tag{3.16}$$

Similarly from (rectangular quasi metric space 2), we can obtain

$$\begin{aligned} d(x_{m(i)-1}, x_{n(i)-1}) + d(y_{m(i)-1}, y_{n(i)-1}) \leq\ & [d(x_{m(i)-1}, x_{m(i)}) \\ & +d(y_{m(i)-1}, y_{m(i)})] \\ & +[d(x_{m(i)}, x_{n(i)}) + d(y_{m(i)}, y_{n(i)})] \\ & +[d(x_{n(i)}, x_{n(i)-1}) \\ & +d(y_{n(i)}, y_{n(i)-1})]. \end{aligned} \tag{3.17}$$

Having the limit as $i \to \infty$ in (3.17), we get by (3.10), (3.11), (3.16)

$$\begin{aligned} \lim_{i\to\infty}[d(x_{m(i)}, x_{n(i)}) + d(y_{m(i)}, y_{n(i)})] =\ & \lim_{i\to\infty}[d(x_{m(i)-1}, x_{n(i)-1}) \\ & +d(y_{m(i)-1}, y_{n(i)-1})]. \end{aligned} \tag{3.18}$$

Applying inequality (3.1) with
$(x, y) = (x_{m(i)-1}, y_{m(i)-1})$, $(u, v) = (x_{n(i)-1}, y_{n(i)-1})$,

$$\begin{aligned} d(x_{m(i)}, x_{n(i)}) &= d(T(x_{m(i)-1}, y_{m(i)-1}), T(x_{n(i)-1}, y_{n(i)-1})) \\ &\leq \varphi(\rho((x_{m(i)-1}, y_{m(i)-1}), (x_{n(i)-1}, y_{n(i)-1}))) \\ &< \rho((x_{m(i)-1}, y_{m(i)-1}), (x_{n(i)-1}, y_{n(i)-1})). \end{aligned} \tag{3.19}$$

Similarly we get

$$d(y_{m(i)}, y_{n(i)}) < \rho((x_{m(i)-1}, y_{m(i)-1}), (x_{n(i)-1}, y_{n(i)-1})). \tag{3.20}$$

Before by adding (3.19) and (3.20) and after taking the limit as $i \to \infty$, we get

$$\begin{aligned} \lim_{i\to\infty}[d(x_{m(i)}, x_{n(i)}) + d(y_{m(i)}, y_{n(i)})] < & \lim_{i\to\infty}[d(x_{m(i)-1}, x_{n(i)-1}) \\ & +d(y_{m(i)-1}, y_{n(i)-1})]. \end{aligned}$$

From (3.18), this is a contradiction. Then $\{x_n\}$, $\{y_n\}$ are Cauchy sequences in rectangular quasi metric space. Because (X, d) is exact there subsist $x, y \in X$ with

$$\lim_{n\to\infty} x_n = x \ and \ \lim_{n\to\infty} y_n = y. \tag{3.21}$$

From continuity of T and since X is Hausdorff, obtaining

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} T(x_n, y_n) = T(x, y),$$

and

$$y = \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} T(y_n, x_n) = T(y, x).$$

□

Corollary 3.2. *Supposing $(X, \preceq)$ is a partially ordered set and (X, d) Hausdorff and exact rectangular quasi metric space. Assuming there is $\varphi : [0, \infty) \to [0, \infty)$ a continuous function such that $\varphi(0) = 0$ and $\phi(t) < t$ for $t > 0$ and also assume $T : X \times X \to X$ is a continuous mapping possessing the mixed monotone property on X,*

$$d(T(x, y), T(u, v)) \leq \varphi(\rho((x, y), (u, v))),$$

for all $x, y, u, v \in X$ for which $x \preceq u$, $y \succeq v$. If there exists $(x_0, y_0) \in X \times X$ with $x_0 \prec T(x_0, y_0)$, $y_0 \succeq T(y_0, x_0)$, at the time T possesses an individual coupled fixed point.

Theorem 3.3. *Suppose $(X, \preceq)$ is partially ordered set and (X, d) Hausdorff and exact rectangular quasi metric space. Assume $T : X \times X \to X$ is a mapping possessing mixed monotone property. Supposing there is a function ψ, ϕ as in Theorem 3.1 and $\psi(t) = 0 \Rightarrow t = 0$. Assuming X has property as below:*

(a) *for all n, if a non-decreasing sequence $\{x_n\} \to x$, at the time $x_n \preceq x$,*

(b) *for all n, if a non-increasing sequence $\{y_n\} \to y$, at the time $y_n \succeq y$.*

If there subsist $x_0, y_0 \in X$ with $x_0 \prec T(x_0, y_0)$ and $y_0 \succeq T(y_0, x_0)$, at the time there subsist $x, y \in X$ with

$$x = T(x, y),\ y = T(y, x),$$

namely T has got an individual coupled fixed point.

Proof. Coming after the proof of Theorem 3.1, constructing a non-decreasing sequence $\{x_n\}$ in X and a non-increasing sequence $\{y_n\}$ in X with $x_{n+1} = T(x_n, y_n)$ and $y_{n+1} = T(y_n, x_n)$ for all $n \geq 0$ and verifying (3.21).

Thence by properties of X, obtaining $x_n \preceq u$ and $y_n \succeq v$ for all $n \geq 0$. By (3.1), having

$$\begin{aligned} d(x_{n+1}, T(x, y)) = d(T(x_n, y_n), T(x, y)) &\leq F(\psi(\rho((x_n, y_n), (x, y))) \\ &\quad \psi(\rho((x_n, y_n), (x, y)))) \\ &\leq \psi(\rho((x_n, y_n), (x, y))). \end{aligned}$$

On letting $n \to \infty$, using (3.21) and properties of φ, we get that

$$\lim_{n \to \infty} d(x_{n+1}, T(x, y)) = 0. \tag{3.22}$$

Conversely, from (regular quasi metric spaces 2) getting

$$d(x, T(x, y)) \leq d(x, x_{n+2}) + d(x_{n+2}, x_{n+1}) + d(x_{n+1}, T(x, y)).$$

While $n \to \infty$ in the foregoing imparity, utilizing (3.21), (3.10) and (3.22), having $d(x, T(x, y)) = 0$ namely $x = T(x, y)$. Analogous, displaying $y = T(y, x)$.

□

4 Conclusion

We have used $C-$Class function on couple fixed point theorems for mixed monotone mappings in exact partially ordered rectangular quasi metric spaces. It is interesting that each fixed point theorems are verifying in the theory.

Competing Interests

Authors have declared that no competing interests exist.

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