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A Possible Exact Solution for the Newtonian Constant of Gravity

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

Compared with our knowledge of other fundamental constants, the exact value of the Newtonian constant of gravity (G) has long been enigmatic, and there is currently no officially accredited exact solution for G. Different from the widely adopted experimental approach and unlike other theoretical ways in resolving the value of G, by applying to the field equation of general relativity two newly developed tensor-based mathematical approaches (one is referred to as "eigen-modulus" to show the converging ability of a tensor, the other is called "the law of tensorial determination" to evaluate indeterminate forms involving tensors), we provide a possible exact solution to G that only relates to the electrical permittivity (ϵ_0) and magnetic permeability (μ_0) of free space, and $G = \frac{1}{16}(\mathrm{m{\cdot}s^{-3}}) \cdot [(\epsilon_0\mu_0)^{\frac{1}{2}}/\eta\pi\ (\mathrm{m^2{\cdot}kg^{-1}{\cdot}s})] = (\epsilon_0\mu_0)^{\frac{1}{2}}/16\pi\eta\ (\mathrm{m^3{\cdot}kg^{-1}{\cdot}s^{-2}})$. η is the corresponding mass density with constant value, with $\eta = 1$ ($\mathrm{kg{\cdot}m^{-3}}$). This research casts doubt on the prevailing hypothesis that G is an independent constant. Our finding may place the theory of gravity and many related researches on a more objective and quantitative footing. The result not only affects the theory of gravity but also plays a key role in maintaining theories of classical mechanics, cosmology, general relativity and astrophysics.

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1 Introduction

Gravity (or gravitation) is a natural phenomenon by which all physical bodies attract one another [1][2]. For such a fundamental interaction, it is worth establishing the corresponding theory to the highest precision [1][2][3]. Among all gravity-related researches, an essential work (as well as one of the several real bottlenecks) is the determination of the Newtonian constant of gravity (also Newtonian gravitational constant, may be referred to as "gravitational constant" for simplicity), G, one of the most fundamental yet elusive physical constants [4][5][6][7][8]. The exact value of this fundamental constant is not only necessary for the accurate determination of the masses of the Earth/planets and the optimization of cosmological and geophysical models [4][6][8], but also may contribute to the development and improvement of the theory of gravity [4][5]. Further, in the context of Newton's law, G is a proportional constant that establishes the connections between all of the quantities in question [1][9]. According to Newton's law of gravity (though Newton never introduced the "gravitational constant"), the gravitational force between two bodies–or particles, identified as P1 and P2–in the universe is [1][9]

$$F = G\frac{m_1 m_2}{r^2}, \tag{1.1}$$

where m_1 and m_2 are the respective masses and r is the distance between their centres of mass. In the context of Einstein's general relativity, G reflects the parameter that space-time responses to a given mass [2][10]. The field equation of general relativity (FEGR, a work of collaborations of Einstein and Grossmann) reads [2][10]

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1.2}$$

where $G_{\mu\nu}$ is the Einstein tensor which is a combination of the Ricci tensor $R_{\mu\nu}$ and the metric tensor $g_{\mu\nu}$, R is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor, c is the speed of light in a vacuum, and $c = (\epsilon_0\mu_0)^{-\frac{1}{2}}$. Given these backgrounds, the exact value of G is widely concerned and has been explored with both experimental approaches [5][6][7][8][11][12][13][14][15][16] and theoretical methods [17][18][19][20][21][22], and these measures are currently under development.

For hundreds of years, G has been measured in the laboratory using increasingly complicated devices aimed at achieving increased precision [5][7][8][11][12][13][14][15]. However, despite three centuries of experimental effort and about 300 custom-designed experiments [3][5][7][8], the accuracy of G has increased relatively little since Cavendish's original experiment in 1798 [5][6][7][8]. The recommended value (2014) by the Committee on Data for Science and Technology (CODATA) is $6.67408(31)\times10^{-11}$ m^3·kg^{-1}· s^{-2} [16]. It should be noted that, people use different systems of units (or, dimensions) in discussing a quantity. Throughout this paper, unless otherwise stated, we will universally and consistently use Standard International (SI) units.

The experimental methods for measuring G include: a torsion pendulum, an atom interferometer, a Fabry-Perot optical cavity, a flexure-strip balance, a falling corner-cube gravimeter, a torsion balance, etc [6][8][11][12][13][14][15]. We will not attempt to evaluate these methods as they are not the focus of this work.

Compared to the effort put into measuring G experimentally, only limited research has focused on determining G theoretically [8][13][17][18][19][20][21][22]. Within the analytical research concerning G, three broad classes of work are performed.

The first set of them predicting the value of G in an empirical way [13][17][18]. Some theoretical estimates of G are conceived within the context of the "standard model" of particles, fields and cosmology, and other estimations that invoke new physics [13][17][18] (all the available estimating results are further shown in table 3. However, none intuition as well as compact expression has been presented in these works due to their lack of systematic theoretical deductions [13][17].

Another, more direct, is introducing G in the definition and/or calculation of other quantities or formulas. Representative examples are as follows: (i) in the two theories that successfully reveal how gravity works, G has first been defined and recognized in Newton's law of gravity and the corresponding formula [1][9], and then been employed in building the field equation of general relativity [2][10]; (ii) the definitions of the Planck mass, Planck length and Planck time have been adopted with the introduction of G [23]; (iii) the "gravitational permeability of free space" is defined and presumed as a constant, and it is $16\pi G/c^2 \approx 3.73 \times 10^{-26}$ m/kg, a tiny effect [24]; (iv) G is widely used in theories of classical mechanics, cosmology, general relativity and astrophysics, whose details will not be enumerated [3][4][8]. More accurately, however, these mathematical derivations related to G are calculations with G rather than achieving the solution to G.

Unlike the previous contributions, Dirac and some other scientists have posited that G may be variable [25][26][27], and this is actually out of the discussion (of this paper) so far.

It should be noted that, in addition to the above-mentioned analytical works, there are considerable researches on the property (characteristic) of G in the framework of tensor analysis in recent years [18][28], though without solution to G attributed to being in its infancy, provide clues (about which kind of mathematical tool should be adopted) to our work.

According to the available materials, until recently, there is no definitive relationship between G and the other fundamental constants, and there is no officially accredited theoretical prediction for its value [6][7][8].

In contrast to the widely employed experimental approach and different from the other theoretical ways, considering the FEGR relates the curvature of space-time to the source (mass), we develop here an analytical solution for G by applying to this equation two tensor-based mathematical approaches that we proposed for the purpose of this work. The first approach is a tensor analysis tool named the eigen-modulus, which is used to show the converging ability of a tensor. The second approach (which we term "the law of tensorial determination") is a technique for evaluating indeterminate forms involving tensors.

This work is arranged as follows. First, we analyzed the problem and its background, explained the key ideas of this paper. These are the main contents in Section 1. The rest of the materials is organized as:

- The majority of our work, "Results," presents the newly obtained exact solution for G in Section 2. The mathematical approach and the performance evaluation experiments are detailed in this section.
- Section 3 selectively details the significant changes (brought by this work) in a variety of physical fields. We also mentioned the methodology contribution of this paper.
- In Section 4, a brief summary is provided.

- Because this work involves extensive mathematical facilities, physical analysis, and further materials (that may be inconvenient for readers to catch the key idea and flow of the paper, if merged in the main text), the supporting information is therefore collected in appendices (Section A to Section E).

2 Results

2.1 The eigen-modulus measure of a tensor

As an important basis of this work, the eigen-modulus measure unfolds an inherent characteristic (or property) called "converging ability," which is previously unknown, of a tensor. The eigen-modulus of a tensor is defined to be the product of the 1-norms of its eigenvalues (see section A for derivations). For example, the eigen-modulus of a rank n (n = 0, 1, 2, ...) mixed tensor $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2\cdots i_k}$, $|T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2\cdots i_k}|_e$, equals to $\prod_{j=1}^{M}|\lambda_j|_1$. Here, λ_j (j =1, 2, 3, ..., M) is the j^{th} eigenvalue of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2\cdots i_k}$, which has a total of M eigenvalues. "$|\cdot|_e$" is the notation for resolving the eigen-modulus measure, while "$|\cdot|_1$" is the operation to resolve the 1-norm. Because an eigenvalue (also principal value or characteristic value) exhibits the properties of the corresponding dimension in a linear transformation, the eigen-modulus reflects the capability of a given tensor (or tensor field) to "converge" its different dimensions. Because the eigenvalues depend only on the tensor, the eigen-modulus is a quantity specified by a given tensor, that is, the eigen-modulus is independent of the choice of coordinate system. This makes resolving the numerical solutions related to tensors feasible.

2.2 The eigen-modulus of the tensors in the FEGR

In the context of the FEGR, the eigen-modulus of the Einstein tensor ($G_{\mu\nu}$) is $1/16$ m$\cdot$s^{-3}, while the eigen-modulus of the energy-momentum tensor (sometimes known as the stress-energy-momentum tensor or stress-energy tensor, $T_{\mu\nu}$) is $\eta[(\epsilon_0\mu_0)^{-\frac{5}{2}}]/8$ kg $\cdot$ m^2 $\cdot$ s^{-5} (see section B for derivations). η is the mass density related to these two interacting bodies (particles), and η = 1 kg$\cdot$m^{-3}. For $G_{\mu\nu}$, space-time curves in a manner consistent with energy [2]. The eigenvalues of $G_{\mu\nu}$ thus relate space-time and matter, dimension to dimension. Therefore, the eigen-modulus of $G_{\mu\nu}$ indicates the ability of the curved space-time to "drive" the matter [2][18]. The bulk properties of matter are described using the energy-momentum tensor [2]. The eigenvalues of $T_{\mu\nu}$ thus reveal the relationship between matter and space-time for each dimension [2][18]. Consequently, the eigen-modulus of $T_{\mu\nu}$ indicates the potential for space-time to be converged, in other words, the ability of matter to curve space-time.

2.3 The law of tensorial determination

Because the eigen-modulus is a determined quantity for a given tensor, our law of tensorial determination states that the numerical result for an indeterminate fraction involving tensors is the ratio of the eigen-modulus of those tensors (for more details on derivations, see section C). This law allows us to resolve an indeterminate fraction involving tensors with some primary calculations. For example, in indeterminate form involving tensors A and B like $\alpha A = B$, the numerical answer (approximation) is $\alpha = |A|_e/|B|_e$.

2.4 The possible solution to the gravitational constant

We now apply the above-mentioned tensor analysis approaches to the FEGR to derive an equation for G. The indeterminate tensorial form for the FEGR can be expressed as $8\pi(\epsilon_0\mu_0)^2 G\,T_{\mu\nu} = G_{\mu\nu}$. Evaluating this indeterminate form yields

$$G = \frac{1}{8\pi(\epsilon_0\mu_0)^2} \cdot \frac{|R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}|_e}{|T_{\mu\nu}|_e} = \frac{1}{8\pi(\epsilon_0\mu_0)^2} \cdot \frac{\frac{1}{16}}{\frac{1}{8}\eta(\epsilon_0\mu_0)^{-\frac{5}{2}}} = \frac{\sqrt{\epsilon_0\mu_0}}{16\pi\eta}. \tag{2.1}$$

Section D has more details on derivations (Appendix D.1) and dimensional analysis (Appendix D.2) related to equation (2.1). Because $\eta = 1$, we define

$$G \simeq \frac{\sqrt{\epsilon_0\mu_0}}{16\pi} \tag{2.2}$$

where "$\simeq$" means "exactly equal in value, while have different units". Generally, equation (2.2) is helpful to have a quantitative knowledge of G before developing qualitative insights into this elusive fundamental constant.

The gravitational constant G is calculated from equation (2.1) to be $6.636046823362696 \times 10^{-11}$ $\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$. We use the following parameter values recommended by the CODATA [16]: $\pi = 3.1415926535897932$, $\epsilon_0 = 8.854187817 \times 10^{-12}$ F·m^{-1} and $\mu_0 = 12.566370614 \times 10^{-7}$ H·m^{-1}. Since ϵ_0 and μ_0 have exact values, the precision of our calculated G only has relationship with the precision of π (with 16 significant figures in this calculation). Because the uncertainty of this calculated G is so tiny and can be ignored in the following experiments, it has not been labelled. We can achieve better precision of the calculated G through admitting more significant figures in π as needed. It is also well known that even the best experiments today can not measure G to more than a few significant figures. The significance of tabulating G to 15 (or more) decimal places is potential precisely tuning of gravity-related research in the future.

2.5 Performance evaluation of our calculated gravitational constant by the field observed data

Our theoretical value of G can now be compared to selected experimentally-observed data. There are two experiments performed: (i) comparing result of equation (2.1) with experimentally-observed data; and (ii) comparing all other available theoretical values of G with experimentally-observed data, which serves as a performance comparison of our result. The experimental results are presented in Fig. 1, table 1 and table 2 (when different gravitational constant results must be distinguished, the observed results are referred to as G_0, and our calculated result is called G_1).

For the absolute difference between the observed results and our result, $|G_1 - G_0|$, the largest is $0.176046823362695 \times 10^{-12}$ m^3·kg^{-1}·s^{-2}, and the smallest is $0.003953176637304 \times 10^{-12}$ m^3·kg^{-1}·s^{-2}. Most of the experimentally-observed results are close to our theoretical value. The best performance of other estimated values is: the maximum absolute difference, $0.2120999999999997 \times 10^{-12}$ $\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$, and the minimum absolute difference, 0×10^{-12} $\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$. Though it may seem that our result and the best one of the other estimated values would have similar performances, detailed studies of them have shown that this need not be the case: (i) previous studies work with supposed phenomenological models that may not be physically reasonable; most of these models have artificially regulated parameters and usually without dimensional analysis, which constrain the technical merits of those results; (ii) at the same time, the observed G values differ from one another by more than one standard deviation [5][6][8][13], reflecting the likelihood that unknown systematic problems exist in the conventional tests [5][6][8][11][12][13][14][15][17][29] and resulting in the current performance evaluation outputs may not be firmly reliable. To explain this in another way: although there is difference between our theory and experiment, without clear evidence, we cannot attribute this discrepancy to possible bias of the experimental values. On the other hand, having not carefully checked and verified the mechanism that results in the discrepancy, it cannot be taken for granted that this discrepancy originates from the theoretical value is an imprecise result.

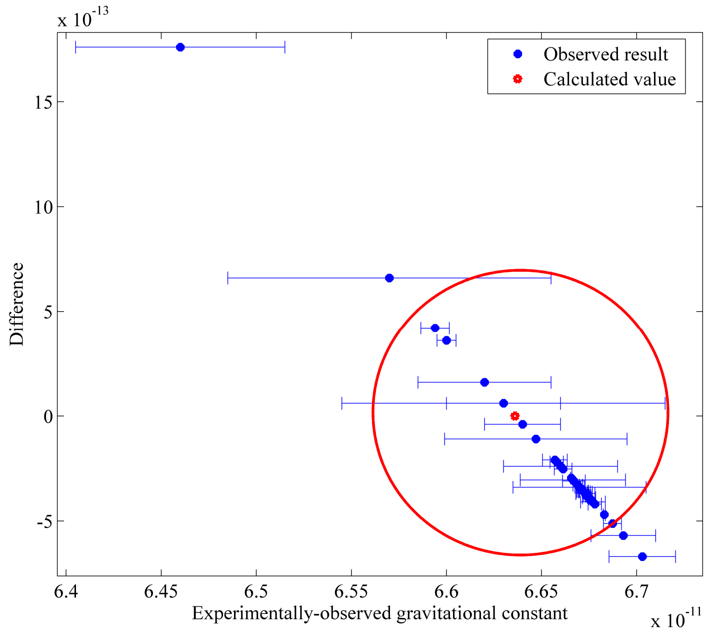

Fig. 1. Difference between calculated gravitational constant (using equation (2.1)) and various observed values. A total of 67 observed results for G were compared to the calculated gravitational constant. Of these, 63 observations are close to the calculated G value with an absolute relative difference smaller than 0.0077. To have an effective conclusion, we used an experimental set-up whose detailed rules are described in section E. The full data set of this experiment is also presented in section E

Table 1. Comparison of all the other available estimated value of the gravitational constant and the observed values in the above-mentioned experiment. The units of the estimated value, maximum absolute difference, minimum absolute difference are "$\times 10^{-11}$ $m^3 \cdot kg^{-1} \cdot s^{-2}$," "$\times 10^{-12}$ $m^3 \cdot kg^{-1} \cdot s^{-2}$," and "$\times 10^{-14}$ $m^3 \cdot kg^{-1} \cdot s^{-2}$," respectively. section E has detailed description of these estimated values of G

Source	Estimated value	Maximum absolute difference	Minimum absolute difference
Bleksley	≈ 9	25.40000000000000	2284.600000000001
Krat	6.67311(4)	2.131099999999999	0.010000000000603
Sternglass	6.6721(5)	2.120999999999997	0
Soldano	6.7340	2.739999999999995	0.186000000000006
Gasanalizade	6.679197926	2.191979259999998	0.011979259999999
Spaniol	6.6725275(9)	2.125275000000003	0.062499999999244
Li	6.67221937(40)	2.122193699999998	0.019370000000519
Naschie	10^{38}	$\approx 10^{39}$	$\approx 10^{41}$

The difference of the last item is a numerical result since this estimated value is dimensionless.

3 Discussion

In a case-by-case way, we hereby discuss some fundamental changes brought by this result, because G is inextricably tied up with theories of mechanics, cosmology, general reality, etc.

With respect to equation (2.1), it can be concluded that G actually has a direct as well as simple relationship with the electrical permittivity and magnetic permeability of free space. The longstanding hypothesis–G is an independent constant within the theory of gravity–may change with a more exact view of the notion presented in equation (2.1). It is noteworthy that this finding may excite the immediate interest of researchers in a broad range of disciplines in science:

(i) The mathematical description of Newton's law of gravity [1][9] is thus stated as

$$F = \frac{(\epsilon_0\mu_0)^{\frac{1}{2}}}{16\pi\eta} \cdot \frac{m_1 m_2}{r^2} = \frac{(\epsilon_0\mu_0)^{\frac{1}{2}} m_1 m_2}{16\pi\eta r^2}. \tag{3.1}$$

Compared with conventional calculation (employing various experimentally obtained G) outputs, the novel way offers us analytical result and improves quantitative characterization of this law. Equation (3.1) can also be written as

$$F \simeq \frac{(\epsilon_0\mu_0)^{\frac{1}{2}}}{16\pi} \cdot \frac{m_1 m_2}{r^2} = \frac{(\epsilon_0\mu_0)^{\frac{1}{2}} m_1 m_2}{16\pi r^2} \tag{3.2}$$

Further, with these preparations, the precise values of mass of the earth and other celestial bodies are expected to be uncovered.

(ii) The field equation of general relativity [2][10] is now

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2\eta} (\epsilon_0\mu_0)^{\frac{5}{2}} T_{\mu\nu}, \tag{3.3}$$

as will feature the relationship between mass and space-time in a deterministic manner which has exact parameters. Equation (3.3) takes another form

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \simeq \frac{1}{2} (\epsilon_0\mu_0)^{\frac{5}{2}} T_{\mu\nu}. \tag{3.4}$$

(iii) Because of the well-defined role of G in many related mathematical models, this result certainly catches attention in several very fundamental ways in modern theoretical studies of gravitational physics, quantum mechanics, particles and fields, and, of course, cosmology. We only provide a brief discussion with equation (2.2).

A representative point is the definition of the Planck scale of mass, length, and time. The Planck mass, Planck length and Planck time have been defined to be $\sqrt{hc/(2\pi G)}$, $\sqrt{hG/(2\pi c^3)}$, and $\sqrt{hG/(2\pi c^5)}$ (h is the Planck constant), respectively [23]. Based on our current knowledge, the Planck mass, Planck length and Planck time are $\sqrt{8hc^2} = 2c\sqrt{2h}$, $\sqrt{h/(32\pi^2 c^4)} = \sqrt{2h}/(8\pi c^2)$, and $\sqrt{h/(32\pi^2 c^6)} = \sqrt{2h}/(8\pi c^3)$ (c can be replaced by $(\epsilon_0\mu_0)^{-\frac{1}{2}}$ which contains only fundamental constants, as is also true in the following texts), where no once undetermined G involved any more, as will contribute to the evolving of physics [30].

The novel result refines the research of "gravitational permeability." Using equation (2.1), we have $16\pi G/c^2 = 1/c^3$ $=3.7114\times10^{-26}$ m/kg, as is consistent with Forward's presumption (according to the original result, $16\pi G/c^2 \approx 3.73\times10^{-26}$ m/kg, the difference here mostly originates from the value of G in Forward's calculation is not an up-to-date result) [24]. We therefore know the so-called "gravitational permeability of free space" should actually be $1/c^3$ m/kg.

In view of the fact that there are many more cross-coupling researches (between G and other scientific disciplines) than that can be discussed in this paper, other closely related notations may be selectively characterized in brief. In de Sitter space of the high energy physics, the parameter of gravitational fluctuations (gravitational coupling constant), κ, is defined as $\kappa^2 = 16\pi G$ [31], which can now be easily resolved by $\kappa^2 = 1/c$. Within research on dark matter and dark energy, the energy density stored on the true vacuum state of all existing fields in the Universe is $\rho_v = \Lambda_0/8\pi G$ [32], wherein Λ_0 is the cosmological constant. ρ_v now has the deterministic form as $\rho_v = 2c\Lambda_0$. In string theory, the string tension in the Planck scale is given by $T_{\mathrm{Pl}} = c^2/G$ [33], which is now converted into $T_{\mathrm{Pl}} = 16\pi c^3$. In cosmology and astrophysics, the expansion rate of the universe, H, is linked to G: $H^2 = (8\pi/3)G\rho_{rad}$, where ρ_{rad} is the energy density [34]. H can now be resolved by $H^2 = \rho_{rad}/6c$.

We should emphasize that, these improvements and those listed in (i) to (ii) not only benefit the calculations but also help us understand the underlying mechanisms in those relations.

(iv) Our finding spontaneously yield clues to how to improve the experimental design of observing G, though precisely measuring G is very hard [3][5][6][7][8]. The relationship in equation (2.1) implies that eliminating spurious forces because of electromagnetic fields may be of obvious importance in those experiments.

Another fundamental question is about the framework of our theory. Conventionally, even without any theoretical proof, people think that G is an independent variable. Our solution is arrived at by applying some proper mathematical facilities to the FEGR which correctly describes the mechanism of gravity. Therefore, our result may be closer to the actual mechanism of the universe.

Based on the above-mentioned materials, it can be shown that, except for the clear discrepancy between this way and the experimental approaches, there are notable differences between our scheme and the other theoretical ways [17][18][19]: we focus our analysis on the FEGR and those tensors relating to gravity (and solution is therefore arrived at), instead of attempting complicated models which may not be scientifically applicable. Hence, the methodology contribution (significance) of our scheme is that it reminds us: to fundamentally uncover physical mechanisms, we should focus efforts on choosing (finding) the suitable mathematical tool to present the underlying principle and validating the result with custom-designed experiments. Further, the two novel tensor-based mathematical approaches are developed to have in-depth knowledge about the FEGR, though, they can also serve as tensor analysis facilities and are not limited to applications in this work.

4 Conclusions

In this study, a theoretical solution to the gravitational constant was obtained. The dimension of this theoretical solution is consistent with that of G given in Newton's law of gravity. This calculated gravitational constant is in close agreement with available measurements. This scheme, if successful, potentially opens the possibility of precisely tuning of many gravity-related calculations. However, there remains small difference between the calculated result and the experimentally-observed data.

According to the solution for the gravitational constant, the wider implication is that gravity and electromagnetism are related in a deeper way, as has been proposed for some time [9][18][35] (and [36] [50] [120][123]). How the two processes interact physically, however, remains to be seen. Another considerable implication is that the possible variation of G may stems from the possible variation of ϵ_0 and/or μ_0. Future research on this topic will need to include further validation of our calculated gravitational constant through the acquisition of more precise experimental observations as well as additional exploration of the relationship between electrical action and the gravitational constant.

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Competing Interests

Author has declared that no competing interests exist.

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APPENDIX

A Definition of the eigen-modulus measure for tensors

As development in tensor analysis occurs, more and more characteristics (properties) of a tensor are unfolded [37][38]. In this section, we will put forward (in detail) the definition of the "eigen-modulus" measure for a tensor. We give the definition of the eigen-modulus measure with a rank (also order or degree) n mixed tensor $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ (n =0, 1, 2, ... When n is either 0 or 1, $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is no longer a mixed tensor). The eigen-modulus measure for a covariant tensor or a contravariant tensor can similarly be arrived at.

A.1 Definition of the eigen-modulus measure for tensor analysis

Definition. An eigen-modulus is a scalar defined over a rank n (n =0, 1, 2, ...) tensor; in a general notation, the eigen-modulus measure is defined as

$$|T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}|_e = \prod\nolimits_{j=1}^{M} |\lambda_j|_1. \tag{A.1}$$

Because eigenvalues are invariants of a tensor, once $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is determined, its eigenvalues are determined. Consequently, $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$'s eigen-modulus is determined accordingly.

Furthermore, if $\lambda_j \neq 0$ $(1 \leq j \leq M, j \in Z)$, it is called a trivial eigenvalue; if $\lambda_j = 0$ $(1 \leq j \leq M, j \in Z)$, it is called a non-trivial eigenvalue.

A.2 Calculation of the eigen-modulus measure for a tensor

The calculation is by case analysis.

Case 1: All the eigenvalues of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is trivial eigenvalue, then the eigen-modulus measure of this tensor can be calculated according to equation (A.1).

Case 2: One or more non-trivial eigenvalues exist in $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$, however, there is at least one eigenvalue of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is trivial eigenvalue. The eigen-modulus measure upon the total eigenvalues of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is therefore a singular solution. In this case, a substituted eigen-modulus measure (non-singular pseudo eigen-modulus) can be arrived at with all the trivial eigenvalue(s) by the following steps.

First, we eliminate all the non-trivial eigenvalues of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$.

Second, the non-singular pseudo eigen-modulus measure of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$, $|T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}|_{\hat{e}}$, can be calculated (with the trivial eigenvalues) with respect to

$$|T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}|_{\hat{e}} = \prod\nolimits_{m=1}^{N} |\lambda_m|_1 \tag{A.2}$$

wherein λ_m (m = 1, 2, 3, ..., N) is the m^{th} trivial eigenvalue of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$, which has a total of N ($N < M$) trivial eigenvalues.

Case 3: All the eigenvalues of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is non-trivial eigenvalue. There is only a singular eigen-modulus measure for this tensor.

A.3 Related topics of the eigen-modulus measure

Dimension of the eigen-modulus measure. According to the definition of the eigen-modulus, its dimension is the product of the dimension of the eigenvalues. For the dimension of an eigenvalue, it equals to the dimension of the corresponding basis of the tensor.

Physical implication of the eigen-modulus measure. In linear transformation, the property of each dimension in a tensor is presented by the corresponding eigenvalue. As a result, according to the definition of the eigen-modulus measure, it shows the ability of a given tensor or tensor field in converging (also attracting) the different dimensions in the tensor. This converging potentiality (also, convergent potential or converging ability) is actually intrinsic and instantaneous.

B The eigen-modulus of the tensorial quantities in the FEGR

In order to obtain the solution to G, we will calculate the eigen-modulus of the tensorial quantities in the FEGR in this section. As described in the main text, the scenario (or, situation) under consideration is two interacting particles (namely, P1 and P2) in the universe. Since the term "particle" is more familiar to us in the discussion related to FEGR, we will use "particle" in the appendices. In the context of this work, "particle" and "body" hold the identical physical implication.

B.1 Rewriting the field equation of general relativity

As we know, $(\epsilon_0\mu_0)^{-\frac{1}{2}}$ equals to the speed of light (c) in a vacuum. In the following mathematical descriptions, we use ϵ_0 and μ_0 instead of c for a variety of reasons: (i) this will facilitate resolving the expression of G with fundamental constants like ϵ_0 and μ_0 with exact value, (ii) this may help further reveal the relationship between a gravitational field, an electric field and a magnetic field, which has long been investigated, and (iii) this expression will make the equation of gravity more comparative with the equation of the electric force. The formula for the FEGR therefore becomes

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(\epsilon_0\mu_0)^2 T_{\mu\nu}. \tag{B.1}$$

The rank of $G_{\mu\nu}$, $R_{\mu\nu}$, $g_{\mu\nu}$, and $T_{\mu\nu}$ is identical 2. The mathematical description can thus be performed in the form of a matrix.

B.2 The eigen-modulus of the Einstein tensor

We will discuss the Einstein tensor according to its components. Because detailing the derivation of this subsection and the next subsection may involve extensive calculations, only the rationale of our deduction and the step-by-step results are presented, on the basis of plenty of references [2][38][39][40].

The Ricci tensor, $R_{\mu\nu}$, is symmetric. At the same time, with respect to Newton's third law, for each entry of $R_{\mu\nu}$, the contributions from P1 and P2 equal in magnitude (absolute value) while maintain opposite signs. As a result, each entry of $R_{\mu\nu}$ is 0 for the system of these two particles [39]. This means

$$R_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{B.2}$$

For the system of interest, the metric tensor, $g_{\mu\nu}$, reads [39]

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{B.3}$$

The Ricci tensor and the metric tensor both hold the relation between space-time related to the two interacting particles. The (0, 0)-entry has the property in length. Each of the (1, 1)-entry, (2, 2)-entry, and (3, 3)-entry maintains the property in inverse-time [40][41][42].

As a consequence, combining (0, 0)-entry and anyone of the (1, 1)-entry, (2, 2)-entry, (3, 3)-entry will result in velocity. Combining (0, 0)-entry and any two of the (1, 1)-entry, (2, 2)-entry, (3, 3)-entry may result in acceleration. The dimensions of the Ricci tensor and the metric tensor are therefore identical (L, T^{-1}, T^{-1}, and T^{-1}).

Like previously described, in this system, the two particles exert a force to each other, with equal magnitude and opposite direction. The Ricci scalar, R, is therefore equals to 1.

Based on the previous work, the Einstein tensor now is

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \tfrac{1}{2} \times \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times 1 \\ &= -\tfrac{1}{2} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}. \tag{B.4}$$

Therefore, $G_{\mu\nu}$'s eigenvalues are $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$. For the Einstein tensor, the value of the eigen-modulus measure is thus $1/16$ m·s^{-3}. According to the definition of the eigen-modulus measure, the dimension of $G_{\mu\nu}$ shows the relation of the space-time in "driving" the matter: one in length, three in inverse time. Both $R_{\mu\nu}$ and $g_{\mu\nu}$ maintain the identical dimension. Consequently, the dimension of $G_{\mu\nu}$ is m·s^{-3} (LT^{-3}). The physical implication of $|G_{\mu\nu}|_e$ has been illustrated in the main text.

B.3 The eigen-modulus of the energy-momentum tensor

For the two particles of interest (m_1 and m_2 are their respective masses), they only interact with each other. This is an extreme scenario in "relativistic particles," which is different from the scenario in "dust" and "fluid" [39]. Since the two particles under consideration is fully relativistic, the energy-momentum tensor has the form [39]

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{13} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix} \tag{B.5}$$

where $T_{ij} = 0$, if $i \neq j$, $i = 0, 1, 2, 3$, $j = 0, 1, 2, 3$. To know the energy-momentum tensor in this interacting system of two particles, the energy density is considered. P1's contribution to the $(0,0)^{\text{th}}$ entry is

$$\sum_{\text{P1}} \frac{m_1 c^2}{V} = \sum_{\text{P1}} \frac{m_1}{\epsilon_0 \mu_0 V} = \frac{\rho_{1,0}}{\epsilon_0 \mu_0} \tag{B.6}$$

where m_1 is the rest mass of P1, and V is the volume of the interaction [39]. $\rho_{1,0}$ is the corresponding rest mass density, $\rho_{1,0} = m_1/V$. Similarly, P2's contribution to the $(0,0)^{\text{th}}$ entry reads

$$\sum_{\text{P2}} \frac{m_2 c^2}{V} = \sum_{\text{P2}} \frac{m_2}{\epsilon_0 \mu_0 V} = \frac{\rho_{2,0}}{\epsilon_0 \mu_0} \tag{B.7}$$

wherein m_2 is the rest mass of P2, and $\rho_{2,0}$ is the corresponding rest mass density, with $\rho_{2,0} = m_2/V$ [39]. Therefore,

$$\begin{aligned} T_{00} &= \sum_{\text{P1}} \tfrac{m_1 c^2}{V} + \sum_{\text{P2}} \tfrac{m_2 c^2}{V} \\ &= \tfrac{\rho_{1,0}}{\epsilon_0 \mu_0} + \tfrac{\rho_{2,0}}{\epsilon_0 \mu_0} \\ &= \tfrac{1}{\epsilon_0 \mu_0} \left(\rho_{1,0} + \rho_{2,0}\right) \end{aligned} . \tag{B.8}$$

Because both P1 and P2 are any unspecified point-like fully interacting objects, the system is with constant mass density $\rho_{1,0} + \rho_{2,0} = 1$ kg·m^{-3} [39][40]. We let η be this constant, and $\eta = 1$ kg·m^{-3}. Thus,

$$T_{00} = \eta (\epsilon_0 \mu_0)^{-1}. \tag{B.9}$$

It should be noted that, in equation (B.9), the dimensionality of mass density (ML^{-3}) is retained for T_{00}.

For the pressure component, T_{11}, it directly relates to the energy over impulse [39]. We consider the contribution from P1

$$\sum_{\text{P1}} \frac{1}{2} \times \frac{\frac{1}{2} m_1 c^2}{m_1 c} = \frac{(\epsilon_0 \mu_0)^{-\frac{1}{2}}}{4} = \frac{1}{4} (\epsilon_0 \mu_0)^{-\frac{1}{2}} \tag{B.10}$$

where the factor 1/2 comes from the fully interacting in two directions (go in opposite directions) [39][40]. Similarly, the contribution from P2 to T_{11} reads

$$\sum_{\text{P2}} \frac{1}{2} \times \frac{\frac{1}{2} m_2 c^2}{m_2 c} = \frac{(\epsilon_0 \mu_0)^{-\frac{1}{2}}}{4} = \frac{1}{4} (\epsilon_0 \mu_0)^{-\frac{1}{2}}. \tag{B.11}$$

So,

$$\begin{aligned} T_{11} &= \tfrac{1}{4} (\epsilon_0 \mu_0)^{-\frac{1}{2}} + \tfrac{1}{4} (\epsilon_0 \mu_0)^{-\frac{1}{2}} \\ &= \tfrac{1}{2} (\epsilon_0 \mu_0)^{-\frac{1}{2}} \end{aligned} . \tag{B.12}$$

Furthermore, the mechanism in T_{22} and T_{33} is identical to the mechanism in T_{11}. It follows, $T_{11} = T_{22} = T_{33}$.

On the basis of the previous analysis, we get

$$T_{\mu\nu} = \text{diag}\left(\eta (\epsilon_0 \mu_0)^{-1},\ \frac{1}{2} (\epsilon_0 \mu_0)^{-\frac{1}{2}},\ \frac{1}{2} (\epsilon_0 \mu_0)^{-\frac{1}{2}},\ \frac{1}{2} (\epsilon_0 \mu_0)^{-\frac{1}{2}} \right). \tag{B.13}$$

Since $T_{\mu\nu}$ is a diagonal matrix, its eigenvalues are $\eta(\epsilon_0\mu_0)^{-1}$, $\frac{1}{2}(\epsilon_0\mu_0)^{-\frac{1}{2}}$, $\frac{1}{2}(\epsilon_0\mu_0)^{-\frac{1}{2}}$, and $\frac{1}{2}(\epsilon_0\mu_0)^{-\frac{1}{2}}$. The dimensions for these eigenvalues are $\mathrm{ML^{-1}T^{-2}}$, $\mathrm{LT^{-1}}$, $\mathrm{LT^{-1}}$, and $\mathrm{LT^{-1}}$, respectively.

According to the definition of the eigen-modulus measure, we have

$$|T_{\mu\nu}|_e = \frac{1}{8}\eta(\epsilon_0\mu_0)^{-\frac{5}{2}}. \tag{B.14}$$

The dimension of $|T_{\mu\nu}|_e$ is $\mathrm{kg{\cdot}m^2{\cdot}s^{-5}}$ ($\mathrm{ML^2T^{-5}}$), while the physical implication of $|T_{\mu\nu}|_e$ has been described in the main text.

C Evaluation of indeterminate form involving different types of variables

In order to get a better handle on tensors, we will study the way to evaluate indeterminate form involving tensors. To facilitate this work, we will shortly discuss evaluation of indeterminate form involving scalar variables. The methodology to evaluate indeterminate form involving tensors is then developed.

C.1 Evaluation of indeterminate form involving only scalar quantities

For functions involving only scalar quantities, L'Hôpital-Bernoulli rule (referred to as "L'Hôpital's rule" in the following text) is the conventional method in evaluating indeterminate forms 0/0 and ∞/∞ [43]. Let, as $x \to p$, the functions $f(x)$ and $\varphi(x)$ be both infinitely small or infinitely large. Then their ratio is not defined at the point $x = p$, and it is said to represent an indeterminate forms 0/0 or ∞/∞, respectively. Fortunately, this ratio may have a limit at the point $x = p$, finite or infinite [43]. In calculus, L'Hôpital's rule uses derivatives to evaluate limits involving indeterminate forms. Application (or repeated application) of this rule usually converts an indeterminate form to a determinate form, allowing easy evaluation of the limit [43][44].

In a basic application, L'Hôpital's rule concerns about the quotient of functions $f(x)$ and $\varphi(x)$. Here, $f(x)$ and $\varphi(x)$ are both differentiable on an open interval I, except possibly at a point p within I.

If

$$\lim_{x\to p} f(x) = \lim_{x\to p} \varphi(x) = 0 \text{ or } \pm\infty, \tag{C.1}$$

and

$$\lim_{x\to p} \frac{f'(x)}{\varphi'(x)} \tag{C.2}$$

exists, and $\lim\limits_{x\to p} \varphi'(x) \neq 0$ for all x in I with $x \neq p$. Here, $f'(x)$ and $\varphi'(x)$ are the derivative of $f(x)$ and $\varphi(x)$, respectively. Then,

$$\lim_{x\to p} \frac{f(x)}{\varphi(x)} = \lim_{x\to p} \frac{f'(x)}{\varphi'(x)}. \tag{C.3}$$

In this way, the differentiation of the numerator and denominator often simplifies the quotient and converts it to a determinate form, facilitating the problem to be more easily resolved [44].

C.2 Evaluation of indeterminate form involving tensors (law of tensorial determination)

As previously described, L'Hôpital's rule is a useful way in evaluating indeterminate forms for functions involving scalar variables. Unfortunately, when considering indeterminate form involving tensors, the L'Hôpital's rule is no longer applicable. The reasons for this point are plentiful. Some important features include the following:

First, the physical implication of the derivative of a tensor is different from the derivative of a real function. On an arbitrary manifold, it is impossible to define a derivative operation on arbitrary tensors in a general and natural way. For differential forms (i.e. alternating covariant tensors), however, this is possible [37][38][45]. The covariant derivative of a tensor is viewed as the orthogonal projection of the Euclidean derivative along a tangent vector onto the manifold's tangent space [45]. This attribute results in the covariant derivative of a tensor is not suitable for the application of the L'Hôpital's rule.

Second, for a rank n tensor, its covariant derivative is a rank $n + 1$ tensor. Application of the L'Hôpital's rule complicates the problem. This makes the L'Hôpital's rule unable to be applied in the calculation.

Given these facts, we must develop novel approaches to evaluate indeterminate form involving tensors.

Law of tensorial determination. For indeterminate form involving tensors A and B like A/B, the numerical answer (approximation) is $|A|_e/|B|_e$. There are four points that apply to this rule:

If $|A|_e|B|_e \neq 0$, $|A|_e/|B|_e$ is the trivial numerical solution for A/B.

If $|B|_e \neq 0$, and $|A|_e = 0$, the trivial numerical solution for A/B is 0.

If $|B|_e = 0$, and $|A|_e \neq 0$, there is no trivial numerical solution for A/B. Let $|B|_{\hat{e}}$ be the non-singular pseudo eigen-modulus for B, then $|A|_e/|B|_{\hat{e}}$ is the non-trivial numerical solution for A/B.

If $(|A|_e)^2 + (|B|_e)^2 = 0$, there is still no trivial numerical solution for A/B. Suppose that $|A|_{\hat{e}}$ and $|B|_{\hat{e}}$ are, respectively, the non-singular pseudo eigen-modulus for A and B, then $|A|_{\hat{e}}/|B|_{\hat{e}}$ is the non-trivial numerical solution for A/B.

Explanation for the law of tensorial determination. Suppose that all the eigenvalues of A and B is trivial eigenvalue, the rationality for the law of tensorial determination is as follows.

First, an eigenvalue is an invariant of a tensor.

Second, each of the eigenvalue characterizes the property in the corresponding dimension for the tensor of interest. At the same time, the property in each dimension regarding the tensor of interest is presented by the corresponding eigenvalue.

Third, the eigen-modulus of a tensor scales the power of the "converging potentiality" of the tensor of interest. Since all the eigenvalues of A and B is trivial eigenvalue, then $|A|_e/|B|_e \neq 0$. As a result, $|A|_e/|B|_e$ shows the ratio of A and B in the "converging potentiality." Therefore,

$$\frac{A}{B} = \frac{|A|_e}{|B|_e}. \tag{C.4}$$

Given the previous rules in the law of tensorial determination, if there is non-trivial eigenvalue exists (or, there are more than one non-trivial eigenvalues exist) in the eigenvalues of A and/or B, the calculation is performed with the non-singular pseudo eigen-modulus and the rationality for this law is similar.

D Solution to the gravitational constant

On the basis of the previous results, solution to G is therefore available. The dimensional analysis for this solution is then realized.

D.1 Solution to the gravitational constant

To resolve the gravitational constant with FEGR (more accurately, the numerical form of FEGR), we organize equation (B.1) into

$$|G_{\mu\nu}|_e = 8\pi G(\epsilon_0\mu_0)^2|T_{\mu\nu}|_e. \tag{D.1}$$

According to the previous developed law of tensorial determination,

$$\begin{aligned}\left[8\pi(\epsilon_0\mu_0)^2\right] G &= \frac{|G_{\mu\nu}|_e}{|T_{\mu\nu}|_e} \\ &= \frac{|R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}|_e}{|T_{\mu\nu}|_e}\end{aligned}. \tag{D.2}$$

Given the results of $|G_{\mu\nu}|_e$ and $|T_{\mu\nu}|_e$ in section B.2 and section B.3, substituting these results into equation (D.2) (and reorganizing the expression) gives

$$\begin{aligned}G &= \frac{1}{8\pi(\epsilon_0\mu_0)^2}\cdot\frac{|R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}|_e}{|T_{\mu\nu}|_e} \\ &= \frac{1}{8\pi(\epsilon_0\mu_0)^2}\cdot\frac{\frac{1}{16}}{\frac{1}{8}\eta(\epsilon_0\mu_0)^{-\frac{5}{2}}} \\ &= \frac{\frac{1}{16}\ (\mathrm{m\cdot s^{-3}})}{\pi\eta(\epsilon_0\mu_0)^{-\frac{1}{2}}\ [(\mathrm{kg\cdot m^{-3}})\cdot(\mathrm{m\cdot s^{-1}})]} \\ &= \frac{\sqrt{\epsilon_0\mu_0}}{16\pi\eta}(\mathrm{m^3\cdot kg^{-1}\cdot s^{-2}})\end{aligned}. \tag{D.3}$$

The solution to G is therefore arrived at by equation (D.3).

D.2 Dimensional analysis for the solution to the gravitational constant

To know the dimension of the solution to G presented in equation (D.3), it is better to analyze the dimension in equation (D.2) [46]. On the left hand side of equation (D.2), the dimension of electrical permittivity (ϵ_0) is $\mathrm{L^{-3}M^{-1}T^4I^2}$, while the dimension of magnetic permeability (μ_0) is $\mathrm{LMT^{-2}I^{-2}}$. It is clear that the dimension of $(\epsilon_0\mu_0)^{-\frac{1}{2}}$ is $\mathrm{LT^{-1}}$. On the right hand side of equation (D.2), the dimension of $|G_{\mu\nu}|_e$ is $\mathrm{LT^{-3}}$, while the dimension of $|T_{\mu\nu}|_e$ is $\mathrm{ML^2T^{-5}}$. So

$$\begin{aligned}\text{The dimension of } G &= \frac{\mathrm{L^4}}{\mathrm{T^4}}\times\frac{\mathrm{LT^{-3}}}{\mathrm{ML^2T^{-5}}} \\ &= \mathrm{L^3M^{-1}T^{-2}}\end{aligned}. \tag{D.4}$$

It can be seen that the dimension of G in equation (D.3) results in $\mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$ ($\mathrm{L^3M^{-1}T^{-2}}$), which is the proper dimension of G in Newton's law of gravity.

E Experimental setup and full data set in the experiments

E.1 Experimental setup

Because the experiments are the comparison between different values, the experimental set-up is rule related to the data selection and calculation method in this course.

Rules on selecting the experimentally-observed data employed in our comparison. Except for the first experimentally-observed G in 1798 and some representative historical results for different experimental methods, most (52 of 67) of the observed values are chosen from the experiments performed after the year 1975 (within the last 50 years). All these samples are selected from the most widely used results. We try to collect data covering various experimental setups as more as possible. At the same time, because the recommended value from CODATA is derived from those experimentally-observed values, and is viewed as with comparatively better performance [13], it is accounted for in the comparison.

The way to resolve the difference, the deviation, and the uncertainty. We will demonstrate the related methods with examples. There is a unique way to resolve the difference and the deviation for all the mentioned data. For example, a newly observed data, $6.67191(99)\times10^{-11}$ $m^3{\cdot}kg^{-1}{\cdot}s^{-2}$ [8], the difference is $G_1 - G_0 = 6.636046823362696 - 6.67191$= -0.03586317663730 ($\times10^{-11}$ $m^3kg^{-1}{\cdot}s^{-2}$), and the deviation is $(G_1 - G_0)/G_1$ =-0.03586317663730/6.636046823362696 = -0.00540429831071.

Because there are two ways to label the uncertainty for the experimentally-observed data, one labels only a single uncertainty and the other labels two uncertainties, the methods to show the uncertainty are therfore different for different data. For sample with one labelled uncertainty, like $6.67191(99)\times10^{-11}$ $m^3{\cdot}kg^{-1}{\cdot}$ s^{-2} [8], the error-bar in Fig. 1 shows the uncertainty of this sample, 99. For sample with two labelled uncertainties, the overall uncertainty is arrived at by the square root of the summation of the uncertainties' square. For example, we have an observed datum (using cold atoms) reads $6.693(27)(21)\times10^{-11}$ $m^3{\cdot}kg^{-1}{\cdot}$ s^{-2} [12], the overall uncertainty is then $(27^2 + 21^2)^{\frac{1}{2}} = 34$. The error-bar of this sample in Fig. 1 thus shows the uncertainty equals to 34.

E.2 The full data set in Fig. 1

The full data set in Fig.1 is presented in table 2

Table 2. The full data set in the experiment (Fig. 1).

No.	Source	Year	G_0	Difference	Deviation
1	Cavendish [47][48]	1798	6.67(7)	-0.033953176637304	-0.005116476351217
2	Reich [49]	1838	6.63(6)	0.006046823362696	0.000911208664383
3	Baily [50]	1843	6.62(7)	0.016046823362695	0.002418129918282
4	Cornu [51]	1873	6.63(17)	0.006046823362696	0.000911208664383
5	Von Jolly [52]	1878	6.46(11)	0.176046823362695	0.026528869980680
6	Boys [53]	1895	6.658(7)	-0.021953176637305	-0.003308170846537
7	Eötvös [54]	1896	6.657(13)	-0.020953176637304	-0.003157478721147
8	Braun [55]	1897	6.658(7)	-0.021953176637305	-0.003308170846537
9	Woodward [56]	1898	6.594(15)	0.042046823362696	0.006336125178422
10	Burgess [57]	1902	6.64(4)	-0.003953176637304	-0.000595712589517

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No.	Source	Year	G_0	Difference	Deviation
11	Heyl [58]	1927	6.6721(73)	-0.036053176637304	-0.005432929814536
12	Heyl [59]	1930	6.670(5)	-0.033953176637304	-0.005116476351217
13	Heyl [60]	1942	6.673(3)	-0.036953176637304	-0.005568552727387
14	Rose [61]	1969	6.674(4)	-0.037953176637304	-0.005719244852777
15	CODATA [62]	1973	6.6720(41)	-0.035953176637303	-0.005417860601997
16	Luther [63]	1975	6.6699(14)	-0.033853176637303	-0.005101407138678
17	Koldewyn [64]	1976	6.57(17)	0.066046823362695	0.009952736187782
18	Sagitov [65]	1979	6.6745(8)	-0.038453176637304	-0.005794590915472
19	Luther [63][66]	1982	6.6726(5)	-0.036553176637305	-0.005508275877231
20	CODATA [62][67]	1986	6.67259(85)	-0.036543176637304	-0.005506768955977
21	Karagioz [68]	1987	6.6731(4)	-0.037053176637304	-0.005583621939926
22	Dousse [69]	1987	6.6722(51)	-0.036153176637304	-0.005447999027075
23	Chen [70]	1989	6.6724(87)	-0.036353176637304	-0.005478137452153
24	Schurr [71]	1991	6.66(6)	-0.023953176637304	-0.003609555097317
25	Zumberge [72]	1991	6.677(13)	-0.040953176637304	-0.006171321228947
26	Schurr [73]	1992	6.6613(93)	-0.025253176637303	-0.003805454860324
27	Oldham [74]	1995	6.669(5)	-0.032953176637304	-0.004965784225827
28	Hubler [75]	1995	6.678(7)	-0.041953176637304	-0.006322013354337
29	Michaelis [76]	1995	6.71540(56)	-0.079353176637303	-0.011957898843922
30	Fitzgerald [77]	1995	6.6656(6)	-0.029553176637304	-0.004453430999501
31	Walesch [78]	1995	6.6719(56)	-0.035853176637304	-0.005402791389458
32	Luo [79]	1995	6.647(96)	-0.010953176637305	-0.001650557467247
33	Bagley [80]	1997	6.67398(7)	-0.037953176637304	-0.005719244852777
34	Karagioz [81]	1998	6.6729(5)	-0.036853176637305	-0.005553483514848
35	Schurr [82]	1998	6.6754(15)	-0.039353176637304	-0.005930213828323
36	Schwarz [12]	1998	6.6873(94)	-0.051253176637304	-0.007723450120464
37	CODATA [83][84]	1998	6.673(10)	-0.036953176637304	-0.005568552727387
38	Luo [85]	1998	6.6699(7)	-0.033853176637303	-0.005101407138678
39	Luo [86]	1998	6.6690(16)	-0.032953176637304	-0.004965784225827
40	Fitzgerald [87]	1999	6.6746(10)	-0.038553176637304	-0.005809660128011
41	Fitzgerald [87]	1999	6.6742(7)	-0.038153176637304	-0.005749383277855
42	Richman [88]	1999	6.6830(11)	-0.046953176637303	-0.007075473981287
43	Nolting [89]	1999	6.6749(14)	-0.038853176637304	-0.005854867765628
44	Kleinevoß [90]	1999	6.6735(29)	-0.037453176637303	-0.005643898790082
45	Gundlach [91]	2000	6.674215(92)	-0.038168176637304	-0.005751643659736
46	Quinn [92]	2001	6.67559(27)	-0.039543176637304	-0.005958845332147
47	Stepanov [93]	2002	6.60(1)	0.036046823362696	0.005431972426082
48	Schlamminger [94]	2002	6.67407(22)	-0.038023176637304	-0.005729793301554
49	CODATA [95]	2002	6.6742(10)	-0.038153176637304	-0.005749383277855
50	Kleinevoß [96]	2002	6.67422(98)	-0.038173176637304	-0.005752397120363
51	Armstrong [97]	2003	6.67387(27)	-0.037823176637304	-0.005699654876476
52	Luo [98]	2003	6.6699(7)	-0.033853176637303	-0.005101407138678
53	Baldi [99]	2005	6.675(7)	-0.038953176637303	-0.005869936978167
54	Hu [100]	2005	6.6723(9)	-0.036253176637303	-0.005463068239614
55	Schlamminger [101]	2006	6.674252(122)	-0.038205176637304	-0.005757219268375
56	CODATA [102]	2006	6.67428(67)	-0.038233176637304	-0.005761438647886
57	Dose [103]	2007	6.67414(24)	-0.038093176637304	-0.005740341750332
58	Fixler [11]	2007	6.693(34)	-0.056953176637304	-0.008582395235187
59	Lamporesi [104]	2008	6.667(12)	-0.030953176637303	-0.004664399975047

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No.	Source	Year	G_0	Difference	Deviation
60	Wang [105]	2009	6.6665(554)	-0.030453176637304	-0.004589053912352
61	Luo [106]	2009	6.67349(18)	-0.037443176637304	-0.005642391868828
62	Parks [14]	2010	6.67234(14)	-0.036293176637305	-0.005469095924630
63	CODATA [13]	2010	6.67384(80)	-0.037793176637304	-0.005695134112715
64	Quinn [13]	2013	6.67545(18)	-0.039403176637303	-0.005937748434592
65	Newman [15]	2014	6.67433(13)	-0.038283176637304	-0.005768973254156
66	Rosi [8]	2014	6.67191(99)	-0.035863176637303	-0.005404298310712
67	CODATA [16]	2014	6.67408(31)	-0.038033176637304	-0.005731300222808

Herein, G_0 is with 10^{-11} m^3·kg^{-1}· s^{-2}. "Difference" is the difference between the observed results and our result, $G_1 - G_0$ ($\times 10^{-11}$ m^3·kg^{-1}· s^{-2}), while "Deviation" is $(G_1 - G_0)/G_1$.

It should be noted that, in table 1, the last 9 digits of $|G_1 - G_0|$ are 317663730, 682336269 or 682336270. This is because that there are 9 to 13 more digits in G_1 than G_0. The subtraction and the precision in computer system lead to this result.

E.3 Information on other estimated values of G

Though seeking the solution for G is definitely very hard, some scientists offered interesting initial results. All the available estimated values of G are collected in table 3. It can be seen that each of these methods asserts that G has a basis out of gravity, which results in impossible to review these techniques within the appendix materials. It may be strange that Naschie's result is much larger than the others, since it is a "classical dimensionless gravitational constant of Newton".

Table 3. The available theoretical values of G.

No.	Source	Physical basis	Estimated value ($\times 10^{-11}$m^3·kg^{-1}·s^{-2})
1	Bleksley [108]	Law of conservation-of-energy applied to an expanding universe	≈ 9
2	Krat [109]	Evaluating elementary particle parameters with fundamental field theory	6.67311(4)
3	Sternglass [110]	A model of "charmonium-like" massive charge pair in early universe scenario	6.6721(5)
4	Soldano [111]	A causal reference frame dependency in the fine structure constant	6.7340
5	Gasanalizade [21]	Ratio of gravitational red shift of H in solar spectrum to electron Compton wavelength	6.679197926
6	Spaniol [22]	Ratio of "rest mass of electron from field self-energies" and Plank constant	6.6725275(9)
7	Li [20]	The "hyperfine splitting" of the ground energy of hydrogen atoms	6.67221937(40)
8	Naschie [112]	Extended renormalizations group analysis for quantum gravity	$\approx 10^{38}$ (without $\times 10^{-11}$, and dimensionless)

F Extensive references related to G

Because the research related to Newtonian gravitational constant is widely concerned, we here list some interesting references on this topic, presumed to benefit scientists in this field. These works are: (i) discussions on experimentally-observed G [107][113][114][115][116][117][118][119]; (ii) theoretical works about G [120][121][122][123]; and, (iii) the possible variation of G [124][125][126].

G Nomenclature

G.1 Quantities related to gravity

F is the gravitational force. m_1, m_2 are the respective masses of P1 and P2. r is the distance between their (P1 and P2) centres of mass. G is the Newtonian constant of gravity.

$G_{\mu\nu}$ is the Einstein tensor. The Ricci tensor is $R_{\mu\nu}$. The metric tensor is $g_{\mu\nu}$, while the Ricci scalar is R. $T_{\mu\nu}$ is the energy-momentum tensor.

c is the speed of light in a vacuum

G.2 Quantities related to tensor analysis

$T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is a rank n mixed tensor ($n = 0, 1, 2, ...$). If $n \geq 2$, it is set as a mixed tensor. When n is either 0 or 1, $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$ is no longer a mixed tensor.

$|T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}|_e$ is the eigen-modulus of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$. Here, "eigen-modulus" is a newly proposed measure for tensor analysis.

λ_j is the j^{th} eigenvalue of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$.

$|T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}|_{\hat{e}}$ is the non-singular pseudo eigen-modulus of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$, if one or more non-trivial eigenvalue(s) exist in the eigenvalues of $T^{i_{k+1}i_{k+2}\cdots i_n}_{i_1 i_2 \cdots i_k}$. Here, "non-singular pseudo eigen-modulus" and "non-trivial eigenvalue" are two newly developed measures.

G.3 Quantities related to evaluation of indeterminate expressions

$f(x)$ and $\varphi(x)$ are two example functions in describing the L'Hôpital-Bernoulli rule.

$f'(x)$ and $\varphi'(x)$ are the derivative of $f(x)$ and $\varphi(x)$, respectively.

A and B are example tensors to illustrate the law of tensorial determination. Here, "law of tensorial determination" is a newly proposed law for tensor analysis.

$|A|_{\hat{e}}$ and $|B|_{\hat{e}}$ are, respectively, the non-singular pseudo eigen-modulus for A and B, if one or more non-trivial eigenvalues exist in the corresponding eigenvalues.

G.4 Temporal variables

It should be noted that, there are some temporal variables (M, i, j, etc.) defined to facilitate the mathematical description. They are not collected here.

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