British Journal of Mathematics & Computer Science
19(3): 1-12, 2016, Article no.BJMCS.29215
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

Mayer's Formula for Black Hole Thermodynamics in Constant Magnetic Field

L. Dahbi[1*] and M. T. Meftah[2]

[1] *LRPPS Laboratory, University of Kasdi Merbah Ouargla, 30000, Algeria.*
[2] *Department of Physics, Faculty of Sciences, LRPPS Laboratory, University of Kasdi Merbah Ouargla, 30000, Algeria.*

Authors' contributions

This work was carried out in collaboration between both authors. Authors LD and MTM studied, performed the work, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/29215
Editor(s):
(1) Carlo Bianca, Laboratoire de Physique Thorique de la Matire Condense, Sorbonne Universits, France.
(2) Kai-Long Hsiao, Taiwan Shoufu University, Taiwan.
(3) Paul Bracken, Department of Mathematics, The University of Texas-Pan American Edinburg, TX 78539, USA.
Reviewers:
(1) P. A. Murad, Morningstar Applied Physics Inc., LLC, Vienna, Austria.
(2) Zhong-Wen Feng, University of Electronic Science and Technology of China, Chengdu, China.
(3) R. Praveen Sam, JNTUA, Anantapuramu, India.
Complete Peer review History: http://www.sciencedomain.org/review-history/16683

Received: 29th August 2016
Accepted: 16th September 2016
Published: 26th October 2016

Original Research Article

Abstract

Aims/ objectives: Using the concepts of the classical thermodynamics, we calculated in this work, the thermodynamic potential of black holes in the presence of a constant externel magnetic field present in the surrounding black hole. This calculation takes into account the developments previously provided by (Hawking, Bekenstein, Davies and Straumann) on the equations governing the dynamics of black holes,with an arbitrary value of the surface gravity κ. Like the four classical thermodynamic Maxwell equations, we have developed, for black holes, new twenty four fundamental equations. Among these equations, particular attention was given to the calculation of specific heats $C_{\Omega,\Phi,M}$ and $C_{J,Q,B}$ and the Mayer formula for a black hole in the presence of magnetic field.

*Corresponding author: E-mail: mewalid@yahoo.com

Keywords: *Surface gravity; event horizon; black hole thermodynamics; thermodynamic potentials; magnetic field; specific heat; Mayer's formula.*

1 Introduction

Black hole refers to hypothetical objects that astronomers have first imagined existed from the theory of relativity and quantum theory. The discovery of pulsars identified with neutron stars, whose existence had also been predicted by theory, made more credible the "black holes" and gave the signal for a real hunt for these strange stars, of much more difficult to detect their main property - if they existent- is to emit no light and no electromagnetic radiation. In an ordinary star, the gravitational force that pulls the material toward the center of the star is balanced by the pressure of the hot gas of the central parts. When the star has consumed all its hydrogen, the thermal pressure is negligible and there is no longer a quantum pressure, especially due to the electrons under the Pauli exclusion principle, can not be too close to each other. If the mass of the star is of the order of that of the sun, then it is the white dwarf status. If the mass is much higher, the pressure of electrons can not balance the gravitation: electrons are absorbed by the protons and, after a transformation may be explosive, there remains a superdense star formed mainly by neutrons. The theory shows that beyond a mass equal to two or three times the solar mass, the neutron star itself is unstable and continues to contract. When the star mass μ reaches to occupy a region that size is less than the gravitational radius $2G\mu/c^2$, the black hole is formed. The creation of such object is accompagnied by the formation of a non-trivial causal structure in the space-time [1], and the best way, to study the geometrical and physical properties of such system, is inevitably the use of the general relativity theory. The space of the black hole is in a state of collapse, and a light beam that falls will not get out. An object of this kind is by definition impossible to see: it is a black hole in the cosmos. In the cosmology theory and the general theory of relativity, the black hole is defined as a region of space-time showing a strong gravitational pull such as no particle or radiation can escape from it. In the framework of this theory, it is checked that a gravitationally collapsing star of mass μ will shrink, in short time measured by an observer on the surface, about a radius of magnitude equal to $2G\mu/c^2$. This radius is commonly known as Schwarzschild radius of the black hole, at which the gravitational field becomes so strong that no further radiation or anything else can escape to infinity. The boundary of the region from which no escape is possible is called the event horizon. Sometimes, black holes can be considered like ideal black bodies, as they reflect no light. Moreover, quantum field theory in curved space-time predicts that event horizons emit Hawking radiation, with the same spectrum as a black body of a temperature inversely proportional to its mass. The theoretical foundations of the subject of black hole dynamics were laid by [2],[3], [4], and the systematic complete treatment is given by [5] and later by [6] [7]. The state equation for the black hole, was given by [8], [9] using the pressure and the volume in the first law of thermodynamics. Later, [10] has corrected the first law of black hole thermodynamics. Hence, it is possible to develop black hole dynamic potentials using the black hole parameters ($\mu,\ J,\ Q, M, \kappa,\ \Omega\ ,\Phi$ and B, the black hole mass, the angular momentum, the electric charge, dipolar magnetic momentum, the surface gravity (the geometric surface at which we have the gravity κ is constant at all), the angular velocity, the electric potential and the magnetic field). The thermodynamics have been studied by many authors [2-7] but they not introduced the magnetic field near boundary of the black hole. In this work, we have developed formula for the specific heats [11] in considering the presence of the constant magnetic field near boundary of the black hole.

Then, we have organized this paper as follows: in section 2 we summarize the main parameters usefull to describe the black hole thermodynamics. Section 3 is devoted to derive the thermodynamic potentials for the black hole with the surrounding externel magnetic field: the internal energy, the free energy (Helmholtz energy), the enthalpy energy and Gibbs energy. For each potential, we have derive, like the Maxwell equations in classical thermodynamics, six relations between the physical quantities for the black hole. A partiular attention was paid to derive the Mayer relation between the specific heats $C_{\Omega,\Phi,M}$ and $C_{j,Q,B}$. A conclusion and a discussion are reported in the section 4.

2 Black Hole Parameters in Magnetic Field

The thermal radiation emitted by a black hole corresponds to a temperature given by [8],[9]

$$T_H = \frac{\kappa h}{2\pi k_B c}. \tag{2.1}$$

where h is the Planck constant, k_B is the Boltzmann constant, c is the light velocity in vaccum and κ is the surface gravity of the black hole given by

$$\kappa = \frac{4\pi(r_+ c^2 - G\mu)}{A}. \tag{2.2}$$

where G is the universal gravitational constant, μ is the black hole mass and A is the area of the event horizon of the black hole. r_+ is the radial coordinate of the event horizon given by [12]

$$r_+ = \frac{1}{c^2}(G\mu + \sqrt{G^2\mu^2 - \frac{J^2c^2}{\mu^2} - GQ^2 - \lambda^2 B^2}). \tag{2.3}$$

where μ, J, Q are the mass, the angular momentum and the electric charge of the black hole respectively and B is the constant magnetic field, assumed to be constant, surrounding the black hole in our subject, whereas λ is a coupling constant with $(cm^3/s^2/Gauss)$ unit caracterising the magnetic interaction between the black hole and the external magnetic field.

Beside T_H and κ, the third important thermodynamical parameter of the black hole is the Bekenstein-Hawking entropy given by

$$S_{BH} = \frac{k_B}{4l_{Pl}^2}A. \tag{2.4}$$

such that A is the area of the event horizon of the black hole given by

$$A/4\pi = r_+ c^2 + (J/c\mu)^2$$

and l_{Pl} is the Planck's length expressed by $l_{Pl} = \sqrt{\frac{Gh}{c^3}} \simeq 10^{-33}cm$. Replacing r_+ by its expression (3) in the expression of the area of the event A, we find

$$A = \frac{4\pi G}{c^4}\left[2G\mu^2 - Q^2 + 2\sqrt{G^2\mu^4 - J^2c^2 - G\mu^2Q^2 - \lambda^2\mu^2B^2}\right]. \tag{2.5}$$

For a general Kerr- Newman black hole, the angular rotation Ω is related to the angular momentum J and the area of the event horizon A of the black hole by

$$\Omega = \frac{4\pi J}{\mu A}. \tag{2.6}$$

the potential of the event horizon Φ is given by

$$\Phi = \frac{4\pi Q r_+}{A}. \tag{2.7}$$

Once the parameters, $\mu, J, \Omega, Q, \Phi, \kappa$ and B are defined, we are able to attack the Mayer formula by using the first law (energy conservation) and the second law (entropy) of the classical thermodynamics.

3 Black Hole Thermodynamics Immersed in Magnetic Fiels

3.1 Internal energy and the first thermodynamics law

The interaction of the black hole with the surrounding fields and matter was widely studied by [13], [14] , we focused here on the interaction with a constant magnetic field B. The first law of thermodynamics, in differential form writes as

$$dE = d[\mu c^2] = \frac{\kappa c^2}{8\pi G} dA + \Omega dJ + \Phi dQ - M dB. \tag{3.1}$$

where M is the magnetic dipole of the black hole. We must mention here from the equation (3.1) that expresses the first law of thermodynamics, that the term ($\sim \kappa c^2 dA/G$) is like the term TdS in classical thermodynamics. This is due to the formulas (2.1) and (2.4). As κ is the gravity, its unit is therefore that of the linear acceleration $[m/s^2]$, G unit is $m^3/kg/s^2$ and the unit of the area A is m^2. It is assumed that the black hole is near of the equilibrium states to the differences in the area A of the event horizons, in the angular momentum J, in the charge Q and in the constant external magnetic field B. Using the equation (3.1) as a perfect differential, we find

$$\frac{\kappa c^2}{8\pi G} = \left(\frac{\partial E}{\partial A}\right)_{J,Q,B}. \tag{3.2}$$

$$\Omega = \left(\frac{\partial E}{\partial J}\right)_{A,Q,B} \tag{3.3}$$

$$\Phi = \left(\frac{\partial E}{\partial Q}\right)_{A,J,B}. \tag{3.4}$$

and

$$M = -\left(\frac{\partial E}{\partial B}\right)_{A,J,Q}. \tag{3.5}$$

Since dE is a perfect differential, we derive six relations connecting the different partial derivatives of the parameters of the black hole:

$$\left(\frac{\partial}{\partial J}\left(\frac{\partial E}{\partial A}\right)_{J,Q,B}\right)_{A,Q,B} = \left(\frac{\partial}{\partial A}\left(\frac{\partial E}{\partial J}\right)_{A,Q,B}\right)_{J,Q,B}. \tag{3.6}$$

Using eqs (3.2) and (3.3) we have the first relation

$$\left(\frac{\partial \kappa}{\partial J}\right)_{A,Q,B} = \frac{8\pi G}{c^2}\left(\frac{\partial \Omega}{\partial A}\right)_{J,Q,B}. \tag{3.7}$$

after that, we have

$$\left(\frac{\partial}{\partial Q}\left(\frac{\partial E}{\partial A}\right)_{J,Q,B}\right)_{A,J,B} = \left(\frac{\partial}{\partial A}\left(\frac{\partial E}{\partial Q}\right)_{A,J,B}\right)_{J,Q,B}. \tag{3.8}$$

Using eqs (3.2) and (3.4) we get the second relation

$$\left(\frac{\partial \kappa}{\partial Q}\right)_{A,J,B} = \frac{8\pi G}{c^2}\left(\frac{\partial \Phi}{\partial A}\right)_{J,Q,B}. \tag{3.9}$$

following,

$$\left(\frac{\partial}{\partial Q}\left(\frac{\partial E}{\partial J}\right)_{A,Q,B}\right)_{A,J,B}=\left(\frac{\partial}{\partial J}\left(\frac{\partial E}{\partial Q}\right)_{A,J,B}\right)_{J,Q,B}. \tag{3.10}$$

Using eqs (3.3) and (3.4) we have the third relation

$$\left(\frac{\partial \Omega}{\partial Q}\right)_{A,J,B}=\left(\frac{\partial \Phi}{\partial J}\right)_{A,Q,B}. \tag{3.11}$$

Next, we have

$$\left(\frac{\partial}{\partial B}\left(\frac{\partial E}{\partial A}\right)_{J,Q,B}\right)_{A,Q,J}=\left(\frac{\partial}{\partial A}\left(\frac{\partial E}{\partial B}\right)_{A,Q,J}\right)_{J,Q,B}. \tag{3.12}$$

Using eqs (3.2) and (3.5) we have the fourth relation

$$\left(\frac{\partial \kappa}{\partial B}\right)_{A,Q,J}=-\frac{8\pi G}{c^2}\left(\frac{\partial M}{\partial A}\right)_{J,Q,B}. \tag{3.13}$$

Next, we have

$$\left(\frac{\partial}{\partial Q}\left(\frac{\partial E}{\partial B}\right)_{J,Q,A}\right)_{A,J,B}=\left(\frac{\partial}{\partial B}\left(\frac{\partial E}{\partial Q}\right)_{A,J,B}\right)_{J,Q,A}. \tag{3.14}$$

Using eqs (3.4) and (3.5) we have the fifth relation

$$\left(\frac{\partial M}{\partial Q}\right)_{A,J,B}=-\left(\frac{\partial \Phi}{\partial B}\right)_{J,Q,A}. \tag{3.15}$$

finally we have,

$$\left(\frac{\partial}{\partial B}\left(\frac{\partial E}{\partial J}\right)_{A,Q,B}\right)_{A,J,Q}=\left(\frac{\partial}{\partial J}\left(\frac{\partial E}{\partial B}\right)_{A,J,Q}\right)_{J,Q,B}. \tag{3.16}$$

Using eqs (3.3) and (3.5) we have the sixth relation

$$\left(\frac{\partial \Omega}{\partial B}\right)_{A,J,Q}=-\left(\frac{\partial M}{\partial J}\right)_{A,Q,B}. \tag{3.17}$$

3.2 Black hole free energy in magnetic field

Similarely to the classical thermodynamics, we define the free energy of the black hole as,

$$F=E-\left(\frac{\kappa c^2}{G8\pi}\right)A \tag{3.18}$$

Therefore, for an arbitrary surface gravity κ, angular momentum J, charge Q, and magnetic field B, the differential if the free energy is given by

$$dF=dE-\left(\frac{Ac^2}{G8\pi}\right)d\kappa-\left(\frac{\kappa c^2}{G8\pi}\right)dA=\left(\frac{\kappa c^2}{8\pi G}\right)dA+\Omega dJ+\Phi dQ-MdB-\left(\frac{Ac^2}{G8\pi}\right)d\kappa-\left(\frac{\kappa c^2}{8\pi G}\right)dA$$

or

$$dF=-\left(\frac{Ac^2}{G8\pi}\right)d\kappa+\Omega dJ+\Phi dQ-MdB \tag{3.19}$$

The differential dF is then the amount that happens when a little change on: the surface gravity κ, the angular momentum J, the charge Q and the magnetic field, occurs simultaniously. At this stage, we can develop the calculation in each particular case: for example, when we are at a surface gravity (defined for κ equal constant), then $dF=\Omega dJ+\Phi dQ-MdB$ and similarly for the others variables, but this not the scope of the present work. As in the case of the internal energy, we use the fact that dF is a perfect differential to find the following relations

$$\left(\frac{Ac^2}{G8\pi}\right) = -\left(\frac{\partial F}{\partial \kappa}\right)_{J,Q,B} \tag{3.20}$$

$$\Omega = \left(\frac{\partial F}{\partial J}\right)_{\kappa,Q,B}. \tag{3.21}$$

$$\Phi = \left(\frac{\partial F}{\partial Q}\right)_{\kappa,J,B}. \tag{3.22}$$

$$M = -\left(\frac{\partial F}{\partial B}\right)_{\kappa,J,Q}. \tag{3.23}$$

Furthermore, six relations between the black hole parameters are

$$\left(\frac{\partial \Omega}{\partial Q}\right)_{\kappa,J,B} = \left(\frac{\partial \Phi}{\partial J}\right)_{\kappa,Q,B}. \tag{3.24}$$

$$\left(\frac{\partial M}{\partial Q}\right)_{\kappa,J,B} = -\left(\frac{\partial \Phi}{\partial B}\right)_{\kappa,Q,J}. \tag{3.25}$$

$$\left(\frac{\partial \Omega}{\partial B}\right)_{\kappa,J,Q} = -\left(\frac{\partial M}{\partial J}\right)_{\kappa,Q,B}. \tag{3.26}$$

$$\left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} = -\frac{c^2}{8\pi G}\left(\frac{\partial A}{\partial J}\right)_{\kappa,Q,B}. \tag{3.27}$$

$$\left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} = -\frac{c^2}{8\pi G}\left(\frac{\partial A}{\partial Q}\right)_{\kappa,J,B}. \tag{3.28}$$

$$\left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B} = -\frac{c^2}{8\pi G}\left(\frac{\partial A}{\partial B}\right)_{\kappa,J,Q}. \tag{3.29}$$

3.3 Black hole enthalpy energy in magnetic field

The enthalpy of the black hole has given by,

$$H = E - \Omega J - \Phi Q + MB \tag{3.30}$$

Therefore,

$$dH = dE - \Omega dJ - \Phi dQ + MdB - Jd\Omega - Qd\Phi + BdM. \tag{3.31}$$

or using eq. (3.1) we have

$$dH = \frac{\kappa c^2}{G8\pi}dA - Jd\Omega - Qd\Phi + BdM. \tag{3.32}$$

Hence

$$\left(\frac{\partial H}{\partial A}\right)_{\Omega,\Phi,M} = \frac{\kappa c^2}{8\pi G}. \tag{3.33}$$

$$\left(\frac{\partial H}{\partial \Omega}\right)_{A,\Phi,M} = -J. \tag{3.34}$$

$$\left(\frac{\partial H}{\partial \Phi}\right)_{A,\Omega,M} = -Q. \tag{3.35}$$

$$\left(\frac{\partial H}{\partial B}\right)_{A,\Omega,\Phi} = M. \tag{3.36}$$

Since dH is a perfect differential, six relations between the black hole parameters are given by the following expressions

$$\left(\frac{\partial \kappa}{\partial \Omega}\right)_{A,\Phi,M} = \frac{8\pi G}{c^2}\left(\frac{\partial J}{\partial A}\right)_{\Omega,\Phi,M}. \tag{3.37}$$

$$\left(\frac{\partial \kappa}{\partial \Phi}\right)_{A,\Omega,M} = \frac{8\pi G}{c^2}\left(\frac{\partial Q}{\partial A}\right)_{\Omega,\Phi,M}. \tag{3.38}$$

$$\left(\frac{\partial J}{\partial \Phi}\right)_{A,\Omega,M} = \left(\frac{\partial Q}{\partial \Omega}\right)_{A,\Phi,M}. \tag{3.39}$$

$$\left(\frac{\partial \kappa}{\partial M}\right)_{A,\Phi,\Omega} = \frac{8\pi G}{c^2}\left(\frac{\partial B}{\partial A}\right)_{\Omega,\Phi,M}. \tag{3.40}$$

$$\left(\frac{\partial B}{\partial \Phi}\right)_{A,\Omega,M} = -\left(\frac{\partial Q}{\partial M}\right)_{\Omega,\Phi,A}. \tag{3.41}$$

$$\left(\frac{\partial J}{\partial M}\right)_{A,J,\Phi} = -\left(\frac{\partial B}{\partial \Omega}\right)_{A,\Phi,M}. \tag{3.42}$$

3.4 Black hole gibbs energy in magnetic field

We define the free enthalpy F_g (Gibbs energy) as

$$F_g = H - \frac{\kappa c^2}{8\pi G}A \tag{3.43}$$

Therefore,

$$dF_g = dH - \frac{(d\kappa)c^2}{8\pi G}A - \frac{\kappa c^2}{8\pi G}dA. \tag{3.44}$$

replacing the previous expression of dH, we find

$$dF_g = -\frac{Ac^2}{8\pi G}d\kappa - Jd\Omega - Qd\Phi + BdM. \tag{3.45}$$

Next, we get

$$\left(\frac{\partial F_g}{\partial \kappa}\right)_{\Omega,\Phi,M} = -\frac{Ac^2}{8\pi G}. \tag{3.46}$$

$$\left(\frac{\partial F_g}{\partial \Omega}\right)_{\kappa,\Phi,M} = -J. \tag{3.47}$$

$$\left(\frac{\partial F_g}{\partial \Phi}\right)_{\kappa,\Omega,M} = -Q. \tag{3.48}$$

and

$$\left(\frac{\partial F_g}{\partial M}\right)_{\kappa,\Omega,\Phi} = B. \tag{3.49}$$

Since dF_g is a perfect differential we have the following six relations between the black hole parameters

$$\left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} = \frac{8\pi G}{c^2}\left(\frac{\partial J}{\partial \kappa}\right)_{\Omega,\Phi,M}. \tag{3.50}$$

$$\left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} = \frac{8\pi G}{c^2}\left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,\Phi,M}. \tag{3.51}$$

$$\left(\frac{\partial J}{\partial \Phi}\right)_{\kappa,\Omega,M} = \left(\frac{\partial Q}{\partial \Omega}\right)_{\kappa,\Phi,M}. \tag{3.52}$$

$$\left(\frac{\partial A}{\partial M}\right)_{\kappa,\Phi,\Omega} = -\frac{8\pi G}{c^2}\left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi,M}. \tag{3.53}$$

$$\left(\frac{\partial B}{\partial \Phi}\right)_{\kappa,\Omega,M} = -\left(\frac{\partial Q}{\partial M}\right)_{\Omega,\Phi,\kappa}. \tag{3.54}$$

$$\left(\frac{\partial J}{\partial M}\right)_{\kappa,J,\Phi} = -\left(\frac{\partial B}{\partial \Omega}\right)_{\kappa,\Phi,M}. \tag{3.55}$$

At all, as we have mentioned earlier, we have 24 fundamental black hole dynamic relations given by eqs ((3.7), (3.9), (3.11), (3.13), (3.15), (3.17)), ((3.24)-(3.29)), ((3.37)-(3.42)) and ((3.50)-(3.55)). We now proceed to deduce an equation involving a $\frac{\kappa c^2}{8\pi G}dA$ term, dubbed as the $\kappa - dA$ equation in short. We have

$$dA = \left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} d\kappa + \left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} d\Omega + \left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} d\Phi + \left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} dM \tag{3.56}$$

$$\frac{\kappa c^2}{8\pi G}dA = \frac{\kappa c^2}{8\pi G}\left[\left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} d\kappa + \left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} d\Omega + \left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} d\Phi + \left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} dM\right] \tag{3.57}$$

The specific heat of the black hole at constant, we define it for Ω , Φ and M are constants,

$$C_{\Omega,\Phi,M} = \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} \tag{3.58}$$

we now using the fundamental relations viz., eqs (3.50) and (3.51) we have

$$\frac{\kappa c^2}{8\pi G}dA = C_{\Omega,\Phi,M}d\kappa + \kappa[\left(\frac{\partial J}{\partial \kappa}\right)_{\Phi,M} d\Omega + \left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,M} d\Phi + \left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi} dM] \tag{3.59}$$

Correspondingly,

$$\frac{\kappa c^2}{8\pi G}dA = \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{J,Q,B} d\kappa + \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial J}\right)_{\kappa,Q,B} dJ$$
$$+ \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial Q}\right)_{\kappa,J,B} dQ + \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial B}\right)_{\kappa,J,Q} dB \tag{3.60}$$

Now we define the specific heat at constant J, Q and B the same as

$$C_{J,Q,B} = \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{J,Q,B} \tag{3.61}$$

3.5 Mayer's formula

Using the perfect differential of area $A(\kappa, \Omega, \Phi, M)$ as

$$dA = \left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} d\kappa + \left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} d\Omega + \left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} d\Phi + \left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} dM \quad (3.62)$$

and in expressing the differentials $d\Omega$, $d\Phi$ and dM as functions of κ, J, Q and B

$$d\Omega = \left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial \Omega}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial \Omega}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial \Omega}{\partial B}\right)_{\kappa,J,Q} dB \quad (3.63)$$

$$d\Phi = \left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial \Phi}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial \Phi}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial \Phi}{\partial B}\right)_{\kappa,J,Q} dB \quad (3.64)$$

$$dM = \left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial M}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial M}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial M}{\partial B}\right)_{\kappa,J,Q} dB \quad (3.65)$$

when replacing the three last differentials in the last expression of dA, we obtain

$$\begin{aligned} dA = {} & \left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} d\kappa + \\ & \left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} [\left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial \Omega}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial \Omega}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial \Omega}{\partial B}\right)_{\kappa,J,Q} dB] + \\ & \left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} [\left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial \Phi}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial \Phi}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial \Phi}{\partial B}\right)_{\kappa,J,Q} dB] + \\ & \left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} [\left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial M}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial M}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial M}{\partial B}\right)_{\kappa,J,Q} dB] \\ = {} & [\left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} + \left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} \left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} + \left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} \left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} + \\ & \left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} \left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B}] d\kappa + (..)\, dJ + (..)\, dQ + (..)\, dB \end{aligned} \quad (3.66)$$

comparing the last expression to the following (by identifying the coefficients of $d\kappa$)

$$dA = \left(\frac{\partial A}{\partial \kappa}\right)_{J,Q,B} d\kappa + \left(\frac{\partial A}{\partial J}\right)_{\kappa,Q,B} dJ + \left(\frac{\partial A}{\partial Q}\right)_{\kappa,J,B} dQ + \left(\frac{\partial A}{\partial B}\right)_{\kappa,J,Q} dB \quad (3.67)$$

we can check immediately

$$\begin{aligned} \left(\frac{\partial A}{\partial \kappa}\right)_{J,Q,B} = {} & \left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} + \left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} \left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} + \\ & \left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} \left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} + \left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} \left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B} \end{aligned}$$

or by multiplying both sides of the last relation by $\frac{\kappa c^2}{8\pi G}$, we find

$$\frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{J,Q,B} = \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} + \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M}\left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} + \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M}\left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} + \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi}\left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B} \quad (3.68)$$

Because we have (see formulas 3.50, 3.51 and 3.53)

$$\left(\frac{\partial J}{\partial \kappa}\right)_{\Omega,\Phi,M} = \frac{c^2}{8\pi G}\left(\frac{\partial A}{\partial \Omega}\right)_{\kappa,\Phi,M} \quad (3.69)$$

$$\left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,\Phi,M} = \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \Phi}\right)_{\kappa,\Omega,M} \quad (3.70)$$

$$\left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi,M} = -\frac{c^2}{8\pi G}\left(\frac{\partial A}{\partial M}\right)_{\kappa,\Omega,\Phi} \quad (3.71)$$

$$C_{\Omega,\Phi,M} = \frac{\kappa c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{\Omega,\Phi,M} \quad (3.72)$$

$$C_{J,Q,B} = \frac{c^2}{8\pi G}\left(\frac{\partial A}{\partial \kappa}\right)_{J,Q,B} \quad (3.73)$$

then the Mayer formula for the black hole is given by

$$C_{\Omega,\Phi,M} - C_{J,Q,B} = \kappa\left[\left(\frac{\partial J}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Omega}{\partial \kappa}\right)_{J,Q,B} + \left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Phi}{\partial \kappa}\right)_{J,Q,B} - \left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial M}{\partial \kappa}\right)_{J,Q,B}\right] \quad (3.74)$$

Due to the sign (-) in front of the last term in this relation, the sign of $(C_{\Omega,\Phi,M} - C_{J,Q,B})$ may also to change. This result can affects a main property of the radiation of black holes that states the black hole specific heat to be negative [11]. Using the comparison between formulas (3.66, 3.67) and by equaling the coefficients of the same differential in both formulas, we can also derive three other important relations, relevant to the area of the event horizon:

$$\left(\frac{\partial A}{\partial J}\right)_{\kappa,Q,B} = \frac{8\pi G}{c^2}\left[\left(\frac{\partial J}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Omega}{\partial J}\right)_{\kappa,Q,B} + \left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Phi}{\partial J}\right)_{\kappa,Q,B} - \left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial M}{\partial J}\right)_{\kappa,Q,B}\right] \quad (3.75)$$

$$\left(\frac{\partial A}{\partial Q}\right)_{\kappa,J,B} = \frac{8\pi G}{c^2}\left[\left(\frac{\partial J}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Omega}{\partial Q}\right)_{\kappa,J,B} + \left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Phi}{\partial Q}\right)_{\kappa,J,B} - \left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial M}{\partial Q}\right)_{\kappa,J,B}\right] \quad (3.76)$$

$$\left(\frac{\partial A}{\partial B}\right)_{\kappa,J,Q} = \frac{8\pi G}{c^2}\left[\left(\frac{\partial J}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Omega}{\partial B}\right)_{\kappa,J,Q} + \left(\frac{\partial Q}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial \Phi}{\partial B}\right)_{\kappa,J,Q} - \left(\frac{\partial B}{\partial \kappa}\right)_{\Omega,\Phi,M}\left(\frac{\partial M}{\partial B}\right)_{\kappa,J,Q}\right] \quad (3.77)$$

From the the definition of the black hole entropy (see equation 2.4), we have in the last equations (3.74-76) derived the change of the entropy of the event horizon, when the parameters J, Q, B and their conjugates Ω, Φ, M changes simultaniously with respect to the change of the surface gravity κ, the angular momentum J, the electric charge Q and the magnetic field B.

4 Conclusion

In this work, we have developped the thermodynamics of the black hole. The new is, the calculation of twenty four relations (like the four classical thermodynamic Maxwell equations) between the black hole parameters. This is achieved by defining the thermodynamic potentials for the black hole in the presence of a constant externel magnetic field present in the surrounding black hole. These calculation takes into account the developments previously provided by Bekenstein on the equations governing the dynamics of black holes for which the specific heats are negatives. In the presence of an externel constant magnetic field, we show in this work, that the sign of the difference $C_{\Omega,\Phi,M} - C_{J,Q,B}$ for the black hole can to change and then modifies a main property of the radiation of the black hole.

Acknowledgement

We wish to acknowledge the support of LRPPS laboratory and its director, by offering us the encouragement, and some technical materials for developing this work. We address also our thanks to Professors at the English department for their help to improve the english language of this work.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Frolov VP, Novikov ID. Black Hole Physics: Basic Concepts and New Developments. Springer, Berlin; 1998.

[2] Bardeen JM, Carter B, Hawking SW. The four laws of black hole mechanics. Commun. Math. Phys. 1973;31:161-170.

[3] Hawking SW. Particle creation by black holes. Commun. Math. Phys. 1975;43:199-220.

[4] Bekenstein JD. Generalized second law of thermodynamics in black-hole physics. Phys. Rev.D. 1974;9:3292-3300.

[5] Davies PCW. The Thermodynamic Theory of Black Holes. Proc. Roy. Soc. Lond. 1977;A353:499-521

[6] Straumann N. General relativity with applications to astrophysics. Springer, Berlin; 2004.

[7] Zixu Zhao, Jiliang Jing. Generalized thermodynamic identity and new Maxwell's law for charged AdS black hole. arXiv preprint arXiv:1607.03565; 2016.

[8] Dolan BP. Pressure and volume in the first law of black hole thermodynamics. Class. Quantum Grav. 2011;28(23):5017-39.

[9] Dolan BP. The cosmological constant and the black hole equation of state. Class. Quantum Grav. 2011;28:125020-41.

[10] Meng-Sen Ma, Ren Zhao. Corrected form of the first law of thermodynamics for regular black holes. Class. Quantum Grav. 2014;31. 245014 (10pp)

[11] Lynden-Bell D. Negative Specific Heat in Astronomy. Physics and Chemistry; Physica A. 1999;263:293-404.

[12] Townsend PK. Black Holes; 1997.
Available: http://arxiv.org/abs/gr-qc/9707012

[13] Gibbons GW, Pang Yi, Pope CN. Thermodynamics of magnetized KerrNewman black holes. Phys. Rev. D. 2014;89(044029):1-14.

[14] Astorino M, Compre G, Oliveri R, et al. Mass of Kerr-Newman black holes in an external magnetic field. Phys. Rev. D. 2016;94(024019):1-11.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/16683