British Journal of Applied Science & Technology
9(2): 206-211, 2015, Article no.BJAST.2015.260
ISSN: 2231-0843

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

A Method for Seeking Range of Missing Plane Based on Law of Energy Conservation

Tian-Quan Yun[1*]

[1]*School of Civil Engineering and Transportation, South China University of Technology, Guangzhou, 510641, P.R. China.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJAST/2015/16660
Editor(s):
(1) Grzegorz Zboiński, Institute of Fluid Flow Machinery, Polish Academy of Sciences, Poland.
(2) Jakub Kostecki, Department of Civil and Environmental Engineering, University of Zielona Góra, Poland.
Reviewers:
(1) Anonymous, St. Peterburg State University, Russia.
(2) Anonymous, Hongik University, Korea.
(3) Anonymous, University of Yaoundé I, Cameroon.
Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=1137&id=5&aid=9276

Short Research Article

Received 10th February 2015
Accepted 28th April 2015
Published 16th May 2015

ABSTRACT

A method for seeking range of a missing plane based on Law of energy conservation is suggested. The method starts with the final information of the location(s) of the plane A(x,y,z) provided by satellites. A flying path with most far from an origin in same time interval [0,T] is proved. Suppose that the flight flies in non-powered state since A(x,y,z). By the Law of energy conservation, the problems corresponding to two ranges of velocity are reduced to Riccati equation and Volterra equation respectively. The exact solutions of these equations are found, and the seeking range is calculated. Finally, the relative motion in longitude direction due to self-rotation of earth is added to the calculated range. A simple calculation is given for reference.

Keywords: Law of conservation energy; Riccati equation; Volterra integral equation; non-powered flying; lift-to-drag ratio; self-rotation of earth.

**Corresponding author: E-mail: cttqyun@scut.edu.cn;*

1. INTRODUCTION

Missing flights e.g., MH370 (2014/03/09 1:07am), QZ8501 (2014/12/28 7:24 am) are widely concerned. Seeking missing flight needs cooperation with international community in three steps: 1, tracking flight with satellites and radar stations, 2, determination the range of seeking. 3, pickup parts of the plane. As an example of application of [1], this short note tries to determine a maximum range of the plane possible dropping based on the law of conservation energy.

The law of conservation energy is one of the greatest discovers of 20's century in the world. This law has been widely used in many fields and widely accepted as a criterion for scientific judge on true or false. For example, the law announces that none of numerous designs of perpetual motion machine in centuries becomes true.

Energy method based on principle of work and energy, conservation law of energy, etc., has been widely used in mechanics. Comparing with the differential equation method, energy method is more welcome by users of practical application. They do not care the description of the whole process, i.e., the solution of the differential equation, while they are interested in some special points, e.g., the turning points, the equilibrium points, the maximum, minimum points, etc. Energy method can achieve to these goals easily. Obviously, to seek a missing plane, we are interested in the largest range of the plane possible dropping, not the details of the actual path, so that the method of energy is suited for our goal. At first, according to [2-4], the plane probably exhausted its fuel and dropped in Indian Ocean. We make an assumption that the missing flight flied in non-powered state since A(x,y,z). Then, we prove an assertion that amount numerous paths, the path flying most far from an origin in the same time interval [0,T] must be flying along a straight line, in section 2.

In section 3, two integral equations involve unknown velocity function, i.e., the Riccati equation and Volterra equation, are set up based on the law of conservation energy corresponding to two ranges of velocity. The exact solutions of these equations are found, and the seeking ranges are calculated.

In section 4, a relative motion in longitude direction in time interval [0,T] due to self-rotation of earth is added to the maximum of the two calculated ranges to form the final range. In section 5, a simple calculation is given for reference. Finally, a conclusion is made.

Note that more general models of the plane dropping can be formulated in the case of more general flight phenomena. However, in the case considered in this paper, the model based on the law of conservation of energy appeared to be sufficient.

2. MATHEMATICAL MODEL

2.1 Coordinate System

Let A (x,y,z) be the location of final information on the plane provided by satellites and radar stations in step 1, where coordinates x, y, z are axes along longitudinal, latitudinal and vertical direction respectively.

Let the calculating coordinates of the plane be A (r, θ, h) in step 2, with origin at A(0,0,0) = A (x,y,z) the location of the plane in air. Where the polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, and $h = -z$, the vertical h-axis is in opposite direction with the z-axis.

Suppose that the plane at location A $(r, 0, h)$ flying along a s-axis, has an increase ds at time t, the relations between the s-axis and r-axis, the s-axis and the h-axis are:

$$r = s\cos\alpha, \tag{2-1}$$

$$h = s\sin\alpha \tag{2-2}$$

The relation between components dr, dh and ds is:

$$dr = ds\cos\alpha, \tag{2-3}$$

$$dh = ds\sin\alpha, \tag{2-4}$$

$\alpha = \alpha(t)$ is the angle between the r-axis and the s-axis.

where

$$r = r(t),\ h = h(t).\ t \in [0, T].$$

2.2 Proof of the path of the flight possible dropping with maximum radius, centered at A (0,0,0)

According to the news report from CNN 2014-03-15 that the flight MH370 probably exhausted its fuel dropping in Indian Ocean [2], suppose that the flight flied in a non-powered flying since A(0,0,0) = A(x,y,z).

There are numerous paths of the plane flying. We need not know and we are impossible to know the actual path of the plane did. We are interested in one path that goes most far from A(0,0,0) = A(x,y,z) in the same time interval [0,T].

Let $R = R(t) = \max\int_0^T v_r(\theta,\alpha)dt$ be the maximum radius of the plane dropping, centered at A(0,0,0). Where $\theta = \theta(t)$ is the angle between plan rOh and plan xOh, $\alpha = \alpha(t)$ is the angle between the s-axis and the r-axis in plan rOh. Suppose that the plane flies along the s-axis with velocity $v(t)$ at time t.

Where

$$v_r = v_r(t) = v(t)\cos\alpha\ , \tag{2-5}$$

$$R = \max\int_0^T v_r(\theta,\alpha)dt, \tag{2-6}$$

The necessary condition of R to be an extreme point is $\acute{R} = dR/dt = 0$.

$$dR/dt = \max\int_0^T \left(\frac{\partial v_r}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial v_r}{\partial \alpha}\frac{\partial \alpha}{\partial t}\right)dt = 0, \tag{2-7}$$

v_r is a function of θ and α, (2-7) give:

$$d\theta/dt = 0, \text{ and } d\alpha/dt = 0. \tag{2-8}$$

i.e., $\theta = C_1$. And $\alpha = C_2$, C_1 and C_2 are constants. That means the plane flies along the s-axis in plane rOh with constant angle $\alpha = C_2$ in any time $t \in [0,T]$, i.e., to get a maximum distance from A (0,0,0) in the same time interval [0,T], the flight must fly along a straight line, say a s-axis.

In the following, the plane flying along the s-axis, i.e., $\alpha = C_2$ (constant), is discussed.

3. SET UP VELOCITY EQUATION BY THE LAW OF CONSERVATION ENERGY

For the case of non-powered flying, there are three forces applied to the plane, i.e., the drag F_s, the lift F_L and the gravity force w.

The formulas of drag F_s and lift F_L are varied with velocity $v(t)$ [5] (notice the difference between the following (3-1),(3-2) and (3-18), (3-19)). In the following, since the super-sound speed and sound speed are impossible for a non-powered flying, so that two possible ranges of velocity, i.e., the sub-sonic speed and low speed are discussed.

3.1 The Velocity of the Plane Under Subsonic Speed

In which the formulas of drag F_s and F_L are usually expressed [5] by (3-1) and (3-2)

$$F_s = k_1 v^2 \tag{3-1}$$

$$F_L = k_2 v^2, \tag{3-2}$$

Where $v = v(t)$ denotes velocity along the s-axis at time t; k_1 and k_2 are constants.

These simple formulas are suited for average case with enough accuracy. If one needs complex formulas with more accuracy on drag and lift, one can refer to [6].

$$w = mg \tag{3-3}$$

where m is the mass of the plane; g = 9.81 (m/sec^2) is the gravity acceleration.

According to assumption, the plane flied in non-powered flying in [0,T]. Thus, the mass m keeps constant in [0,T] and the law of conservation energy holds in [0,T].

Drag F_s applies along the s-axis with opposite direction of ds; F_L applies perpendicular to the s-axis. The w applies along the h-axis (z-axis). During the process of the plane flight along the s-axis, its displacement from 0 to s, no work has been done by force F_L, so that the mechanical energy E(t) is:

$$E(t) = mgh - \int_0^s k_1 v^2 ds + (1/2)mv^2, \tag{3-4}$$

Where the first, second and last terms of the right-hand side denote the work done by w, by F_s and the kinetic energy respectively. $h(0) = z$, $h(T) = 0, v = v(t), s = s(t)$.

$$E(0) = mgz + (1/2)mv_0^2, \tag{3-5}$$

By the Law of conservation energy; $E(t) = E(0)$, we have

$$gz + \left(\frac{1}{2}\right)v_0^2 = gh - \frac{k_1}{m}\int_0^s v^2\, ds + \frac{1}{2}v^2(t), \tag{3-6}$$

(3-6) is a non-linear integral equation with unknown functions $h(t)$, $s(t)$ and $v(t)$,

$h(t) = s(t)\sin\alpha$, where $\alpha = C_2$

The solution of integral equation (3-6) for $\alpha = C_2$ (constant).

Differentiating both sides of (3-6) respect to t, we have

$$\frac{dv}{dt} = \frac{k_1}{m}v^2 - g\cos\alpha, \quad (3\text{-}7)$$

(3-7) is a special kind of Riccati function [7].

The solution of this special kind of Riccati function is (formula 1.23 of [3]):

$$v(t) = \frac{\rho\sqrt{AB}+B\,\mathrm{th}\sqrt{AB}(t-\sigma)}{\sqrt{AB}+A\rho\,\mathrm{th}\sqrt{AB}(t-\sigma)}, \quad \text{for } AB > 0, \quad (3\text{-}8)$$

Where $A = -(\frac{k_1}{m})$, $B = -g\cos a$, (ρ, σ) is a point the solution v(t) is passing through.

Where $\rho = v_0$, $\sigma = 0$. Substituting ρ and σ into (3-8), we have

$$v(t) = \frac{v_0\sqrt{AB}+B\,\mathrm{th}\sqrt{AB}\ t}{\sqrt{AB}+Av_0\mathrm{th}\sqrt{AB}\,t}, \quad (3\text{-}9)$$

The angle α can be found by equilibrium equations in the s-axis and in the L-axis, which is perpendicular to the s-axis,

$$\sum F_s = 0,\ F_s - w\ \sin\alpha = 0, \quad (3\text{-}10)$$

$$\sum F_L = 0,\ F_L - w\ \cos\alpha = 0, \quad (3\text{-}11)$$

Substituting (3-1), (3-2) into (3-10) and (3-11), we have

$$\tan\alpha = F_s/F_L = k_1/k_2, \quad (3\text{-}12)$$

Where the coefficients k_1 and k_2 can be found by flight test, by wind tunnel test, for a wished speed. A higher lift-to-drag ratio (L/D ratio) is typically one of the major goals in aircraft design. It leads directly to better fuel economy, climb performance, and glide ratio. According to data [9], the L/D ratio of Boeing 747 in cruise speed is 17.

Now, we can determine all data we want. By energy at time T, (3-4) gives:

$$E(T) = (1/2)mv(T)^2 = (1/2)m\left[\frac{v_0\sqrt{AB}+B\,\mathrm{th}\sqrt{AB}\ t}{\sqrt{AB}+Av_0\mathrm{th}\sqrt{AB}\,t}\right]^2, \quad (3\text{-}13)$$

By the law of conservation energy, E(0) = E(T), we have

$$2gz + v_0^2 = \left[\frac{v_0\sqrt{AB}+B\,\mathrm{th}\sqrt{AB}\ t}{\sqrt{AB}+Av_0\mathrm{th}\sqrt{AB}\,t}\right]^2, \quad (3\text{-}14)$$

$$T = \frac{\sqrt{AB}(v_0-\sqrt{2gz+v_0^2})}{Av_0\sqrt{2gz+v_0^2}\mathrm{th}\sqrt{AB}-B\,\mathrm{th}\sqrt{AB}}, \quad (3\text{-}15)$$

$$S(T) = \int_0^T v(t)dt = \frac{G}{I}\ln\left[\frac{(I+JT)}{I}\right] + H\left[\frac{T}{I} - \frac{J}{I^2}\ln\frac{I+JT}{I}\right], \quad (3\text{-}16)$$

Where

$G = v_0\sqrt{AB}$, $H = B\,\mathrm{th}\sqrt{AB}$, $I = \sqrt{AB}$, $J = Av_0\sqrt{AB}$.

$$R = s(T)\cos\alpha = \left\{\frac{G}{I}\ln\left[\frac{(I+JT)}{I}\right] + H\left[\frac{T}{I} - \frac{J}{I^2}\ln\frac{I+JT}{I}\right]\right\}\cos a, \quad (3\text{-}17)$$

(3-17) shows the largest radius R of the plane possible dropping with centered at A(0,0,0)=A(x,y,z).

3.2 The Plane Flight with Low Velocity

In the case of a low flight velocity, the drag F_s and lift F_L are expressed by (3-18) and (3-19).

$$F_s = k_3 v, \quad (3\text{-}18)$$

$$F_L = k_4 v, \quad (3\text{-}19)$$

Where k_3, k_4 are constants.

Similarly to section 3.1 above,

$$E(t) = mgh - \int_0^s k_3\, v ds + (1/2)\, mv^2, \quad (3\text{-}20)$$

By the law of conservation energy, E(0) = E(t), we have:

$$gz + \left(\frac{1}{2}\right)v_0^2 = gh - \int_0^s \frac{k_3}{m} vds + \left(\frac{1}{2}\right)v^2, \quad (3\text{-}21)$$

(3-21) is a non-linear integral equation with unknown functions h, v, v. Using (2-2) and $v(t) = ds/dt$, (3-21) becomes a non-linear Volterra integral equation of the second kind [8] and can be easily solved.

Differentiating both sides of (3-21) respect to t, we have:

$$dv/dt = \left(\frac{k_3}{m}\right)v - g\cos\alpha, \quad (3\text{-}22)$$

Let $u = \left(\frac{k_3}{m}\right)v - g\cos\alpha$, then, the solution of (3-22) is:
$u = C\exp\left[\left(\frac{k_3}{m}\right)t\right] - g\cos\alpha$, or

$$v(t) = \left(\frac{m}{k_3}\right)\{C\exp\left[\left(\frac{k_3}{m}\right)t\right] - g\cos\alpha\}, \quad (3\text{-}23)$$

$t = 0$, $v(0) = v_0$, we have $C = v_0(\frac{k_3}{m})$ - $g\cos\alpha$, then,(3-23) becomes:

$$v(t) = v_0\exp\left[\left(\frac{k_3}{m}\right\} t\right] - \left(\frac{m}{k_3}\right)\left[\exp\left(\frac{k_3}{m}\right)t - 1 g\cos\alpha, \quad (3\text{-}24)$$

The angle α can be found by equilibrium equations in the s-axis and in the L-axis, which is perpendicular to the s-axis,

$$\sum F_s = 0,\ F_s - w\sin\alpha = 0, \quad (3\text{-}25)$$

$$\sum F_L = 0,\ F_L - w\cos\alpha = 0, \quad (3\text{-}26)$$

Substituting (3-18), (3-19) into (3-25) and (3-26), we have

$$\tan\alpha = F_s/F_L = k_3/k_4, \quad (3\text{-}27)$$

Where the coefficients k_3 and k_4 can be found by wind tunnel tests for low speed.

Now, we can determine all data we want. By energy at time T, we have

$$E(T) = \left(\frac{1}{2}\right)mv^2(T) = \left(\frac{1}{2}\right)m\{v_0\exp\left(\frac{k_3}{m}\right)T - \text{mk3gcosαexpk3mT−1}\}2,$$

By the law of conservation energy, we have

$$E(0) = E(T), \quad (3\text{-}28)$$

$$\frac{2gz}{m} + v_0^2 = \{v_0\exp\left(\frac{k_3}{m}\right)T - \left(\frac{m}{k_3}\right)g\cos\alpha\,[\exp\left(\frac{k_3}{m}\right)T - 1]\}^2, \quad (3\text{-}29)$$

We have

$$T = \left(\frac{m}{k_3}\right)\ln U, \quad (3\text{-}30)$$

$$U = \frac{\sqrt{\frac{2gz}{m}+v_0^2} - \left(\frac{m}{k_3}\right\}g\cos a}{v_0 - \left(\frac{m}{k_3}\right\}g\cos a}, \quad (3\text{-}31)$$

Finally, by (3-24), we find

$$s(T) = \int_0^T v(t)dt = \left(\frac{m}{k_3}\right)\exp\left[\left(\frac{k_3}{m}\right)T\right][v_0 - \left(\frac{m}{k_3}\right)g\cos\alpha], \quad (3\text{-}32)$$

$$R = s(T)\cos\alpha = \left(\frac{m}{k_3}\right)\exp\left[\left(\frac{k_3}{m}\right)T\right][v_0 - \left(\frac{m}{k_3}\right)g\cos\alpha]\cos\alpha, \quad (3\text{-}33)$$

(3-33) shows the largest radius R of the plane possible dropping with centered at A (0,0,0) = A (x,y,z).

4. ADDITIONAL RELATIVE MOTION DUE TO EARTH SELF-ROTATION

In the following, the earth is considered as a sphere with a radius $R_\oplus$ = 6371 km, self-rotating around north-south polar axis with angular velocity $\Delta\emptyset = 360^o/24h = \pi/12$ (radian/hour) = $\pi/720$ (radian/min).

The displacement Δx of point A(x,y,0) in time [0,T] due to self-rotation of earth is:

$$\Delta x = R_\oplus \cos y\,\Delta\emptyset T, \quad (4\text{-}1)$$

Where T represents the time from A(x,y,z) to dropping into ocean. $R_\oplus$cosy represents the radius of the latitude circle at y.

Since the plane is flying in air, not affiliated with the earth, so that the self-rotation of earth does not influence on the plane. However, when the plane dropping into the sea, the calculated dropping point on earth has moved a distance Δx, so that Δx should be added into each boundary point of the calculated range. For simple, Δx is added to the center A (x,y,0), i.e., A (x+Δx,y,0). The final calculated range is a circle, centered at A (x+Δx,y,0) with radius R ((3-17) or (3-33)), depends on the initial velocity v_0 belonging to under sub-sound speed or low speed.

There is expert evaluate that the influence on the final result due to earth self-rotation is about 6%.

If several initial points are provided by satellites, then, repeated the above calculation for each initial points.

5. A SIMPLE CALCULATION OF THE RANGE OF POSSIBLE DROPPING

The above calculation would be complex. One needs a simple estimation for reference.

Suppose that The L/D = 17 = $\cot\alpha$ holds for all altitude in [0,T]. The initial conditions from data [10], the cruising speed and the altitude of a Boeing 747 at $t = 0$ are: 585 mph (or 940 km/h) and 12,000 m respectively. The L/D = 17 = $\cot\alpha$. So the radius $R = 17\,h(0) = 17 \times 12000 = 204{,}000$ m = 204 km. The total time $T = 12/(940\sin\alpha) = 12/(940\tan\alpha) = 0.21$ hour. By (4-1), we have

$$\Delta x = [\pi \times 0.21/12]R_{\oplus}\cos y = 0.05481\times 6371\cos y = 116\cos y.\ (\mathrm{km}).$$

6. CONCLUSION

Suppose that the flight flied in a non-powered flying since A(0,0,0) = A(x,y,z).

There are numerous paths of the plane flying. The path which goes most far from A(0,0,0) must be flying along a straight line in the same time interval [0,T].

Two integral equations involve unknown velocity function, i.e., the Riccati equation and Volterra equation, are set up based on the law of conservation energy corresponding to two possible ranges of velocity. The exact solutions of these equations are found, and the seeking ranges are calculated. In which the angles of depression $\tan\alpha = F_s / F_L$ (3-12) or (3-27), depending on the ratio of drag to lift corresponding to two possible ranges of velocity respectively.

Finally, The final calculated range is a circle, centered at A(x+Δx,y,0) with radius R ((3-17) or (3-33)), depends on the initial velocity v_0 belonging to under sub- sound speed or low speed. We can choose the maximum one of R for seeking.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Yun Q. Criteria of True and Its Applications. European International Journal of Science sand Technology. 2014;3(3):45–48.
2. Sebastian Anthony. Malaysian PM confirms flight MH370 ended in the Indian Ocean., March 24 2014 10:24am. Available:www.extremetech.com
3. The search for flight MH370, BBC news, Asia Available:www.bbc.com/news/world_asia_26514556, 5, September 2014, Last updated 15:22
4. Missing Malysia Plane MH370, What we know, BBC news. Available:www.bbc.com/news/wolrd_asia_26503141 30 January 2015, updated at 15:00
5. Darmofal David L, Lift and Drag, Introduction, 2004-09-09, (in Chinese); 9009, 2012-02-22. Available:www.doc88.com/p
6. Zhu G, Bearman PW, Graham JMR. Prediction of drag and lift of wings from velocity and vorticity. The Aeronautical Journal. 2007;699–704.
7. Kamke E. Differentialgleichungen, Lösungsmethoden und Lösungen 1, Gewöhnliche, Differentialgleichungen. Chinese version, translated by zhang Hong-ling, Scientific Publishers, Beijing, 1977;342.
8. Yun TQ. Integral equations and its application in mechanics, South China University Publishers Press Guangzhou. 1990;50. (in Chinese) ISBN: 7-5623-0151-4.
9. Edit. From Wikipedia, the free encyclopedia, Lift-to-Drag ratio, 2015-03-10.
10. Edit. From Wikipedia, the free encyclopedia, what is the cruising speed of a 747? What is the cruising altitude of a 747? 2015-02-23. Available:www.answers.com/Q

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history.php?iid=1137&id=5&aid=9276