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Computational Analysis for the Dynamical System Associated to an Access Control Structure

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Abstract

This paper presents some analytical considerations regarding the dynamical behavior of an access control structure, based on the mathematical model associated to this structure. This structure type is largely analyzed in the literature. A modern approach of this structure based on SMA (shape memory alloy) is taken into account, because of some particular advantages: unique characteristics (superelastic effect, as well as the single and double shape memory effects), damping capacity of noise and vibration, resistance to fatigue, diversification of the control and command possibilities.

The basic aim is the qualitative analysis of the mathematical model associated to this structure. Namely, the dynamic system associated to the variation of the angle q describing the position of the access control structure is analyzed from the influence of parameters standpoint. The MAPLE11 soft is used in order to evaluate the behavior of the equation solution with respect to the parameters variation.

This analysis produces a data collection which is useful both for further developing a fuzzy logic controller for the active control of this access structure and for further refinements of the mathematical model associated to this structure type.

Keywords: Discrete-time systems; automated systems; sensitivity; simulation.

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1 Introduction

Memory Alloys (SMAs) are a class of smart materials that possess the ability to undergo shape change at low temperature and retain this deformation until they are heated, at which point they return to their original shape. SMAs present typical thermo-mechanical behaviors, like pseudo-elasticity and shape memory effects (one-way and two way). The cause is a martensitic phase transformation between a high temperature parent phase, austenite (A), and a low temperature phase, martensite (M). In the absence of stress, the start and finish transformation temperatures are denoted Ms, Mf (martensite start and martensite finish) and As, Af (austenite start and austenite finish). Due to their “smart properties”, these materials are widely used in practice. The structures using them have mechanical and impact tests, and also static and dynamic analysis [1].

SMA related design is not easy. Several aspects must be considered before the final prototype takes place. One of the major obstacles to overcome are the intertwined properties of shape memory alloys. Most of the physical, electrical and mechanical aspects of shape memory depend on each other and at some point design decisions must be made to reduce the number of variables. Some of these correlations are: Force vs. Cycle Times vs. Power; Stroke vs. Durability vs. Envelope Volume; Control Aspects and viability [2,3].

The SMA springs work as linear actuators by contracting with great strength and speed when heated. These springs actuators can be attached to barrier structures, and can be activated to switch positions of the barrier. The active shape-change control of SMA spring can effectively increase the efficiency of such a barrier at several different regimes. SMAs exhibit a large temperature dependence on the material shear modulus, which increases from low to high temperature. Therefore, as the temperature is increased the force exerted by a shape memory element increases dramatically. Consequently, the determination of the transformation temperatures is necessary to establish the real shear modulus values at these functional temperatures for a high-quality design of SMA elements [4,5].

In order to determine the required transformation temperatures of SMA spring, the Differential Thermal Analysis (DTA) and Differential Scanning Calorimetry (DSC) methods were used [6]. Also, Thermogravimetric Analysis (TG) was used to prove the stability of the alloys.

Starting from the dynamic calculus of the access control structure, in what follows there is realized an analytical evaluation and computational testing for the mathematical model associated to this structure, using MAPLE11 software. It is presented a particular type solution of the differential equation associated to the model, and the behavior of its trajectory, too. This would help understanding the experimental standpoint: taking into account that the SMA spring must develop, at high temperature, the necessary force to lift the barrier arm by S=12 mm, the question is: *how large the intensity in spring can be, in order to rise the access arm in an imposed time?* The data obtained will be used for refining the experiments in new conditions, on one hand, and, on the other hand, will allow new qualitative approaches for the associated mathematical model.

2 Methodology

2.1 Dynamic Calculus for the Access Control Structure with SMA Actuator

In Fig. 1 is exhibited the mechanical model of the experimental arrangement simulating the barrier. The force is ensured by the SMA spring. At least for the beginning of a certain motion of the barrier starting from an resting position, the direction of this force can be considered as being vertical.

The nickel titanium alloys, used in the present research, generally referred to as Nitinol, have compositions of approximately 50 atomic %Ni/ 50 atomic % Ti.

The features of the model are the following:

- We consider as known the ratio $k=AO_0/AB$
- The length of the barrier arm AB is L. The center of mass of this arm is noted C. We have AC = CB
- The barrier arm is considered as being kind of homogenous and rigid. Its mass is noted m_1
- The external concentrated mass attached on the arm at its bound B is m_2
- We consider the case that k > 0 and k < 0.5
- The current position of the arm is described by the generalized coordinate q

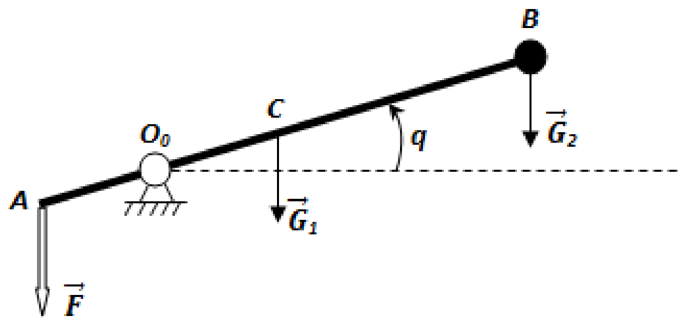

Fig. 1. Mechanical model of the experimental arrangement

Both the dynamic calculus and the mathematical model are based on the Hamiltonian mechanics principles. There are taken into account:

- The arm gravity force;
- The gravity force of the external weight (due to the mass m_2);
- The SMA spring force, applied in A

The equilibrium condition for the arm is obtained in its vertical position, by applying the "δ-Hamilton" variational principle. The case $q=\frac{\pi}{2}$ describes a stable equilibrium status when:

$$m_1\cdot g\cdot L\cdot\left(\frac{1}{2}-k\right)+m_2\cdot g\cdot L\cdot(1-k)<k\cdot L\cdot F \tag{1}$$

Also, the case $q=\frac{3\pi}{2}$ describes a stable equilibrium position if

$$m_1gL\left(\frac{1}{2}-k\right)+m_2gL(1-k)>kLF \tag{2}$$

The system will develop some motion, but this motion will not be kind of uniform. We can study this motion using, briefly, the Lagrange method. The kinetic energy of the system, elementarily calculated, is:

$$T=\dot{q}^2\cdot\frac{m_1\cdot L^2}{2}\cdot\left[\frac{1}{3}+k\cdot(k-1)\right]+\dot{q}^2\cdot\frac{m_2\cdot L^2}{2}\cdot(1-k)^2 \tag{3}$$

Of course we have $\frac{\partial T}{\partial q}=0$ which implies

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{q}}\right)=\ddot{q}\cdot m_1\cdot L^2\cdot\left[\frac{1}{3}+k\cdot(k-1)\right]+\ddot{q}\cdot m_2\cdot L^2\cdot(1-k)^2 \tag{4}$$

Taking into account the generalized force Q [7], the mathematical model of the motion will result from applying the well-known Lagrange equality

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = Q \qquad (5)$$

Thus, the following differential equation is obtained:

$$\ddot{q} \cdot \left\{m_1 \cdot L^2 \cdot \left[\frac{1}{3} + k \cdot (k-1)\right] + m_2 \cdot L^2 \cdot (1-k)^2\right\} = \cos q \cdot \left[kLF - L \cdot \left(\frac{1}{2} - k\right) \cdot m_1 \cdot g - L \cdot (1-k) \cdot m_2 \cdot g\right] \qquad (6)$$

Let us denote:

$$\lambda = \frac{kLF - L \cdot \left(\frac{1}{2} - k\right) \cdot m_1 \cdot g - L \cdot (1-k) \cdot m_2 \cdot g}{m_1 \cdot L^2 \cdot \left[\frac{1}{3} + k \cdot (k-1)\right] + m_2 \cdot L^2 \cdot (1-k)^2} \qquad (7)$$

We have obviously λ=const. and, in the announced conditions, $\lambda > 0$ [7]. Then the mathematical model becomes:

$$\ddot{q} - \lambda \cdot cosq = 0 \qquad (8)$$

2.2 Mathematical Approach. Methodology

The basic aim is to realize a computational testing of the differential equation (8) associated to the access control structure, in order to get an analytical standpoint for the model and to use it in further analysis. The study is in fact on an automated system studied in discrete time, which is based on the variation of the arm angle in time, with respect to the parameters.

The methods are designed in two categories: analytical and computational. It is used the MAPLE11 soft to analyze the solution of the equation (8), with initial conditions imposed by experiment. This soft has a lot of fast appliances both for solving and graphical analyzing differential equations [8].

2.2.1 Computational approach

It was used the "*dsolve*" procedure for getting the solution of the Cauchy problem associated to the model. The solution was asked in series form, taking into account a future aim of getting some approximate evaluations.

As a general ordinary differential equation (ODE) solver, "*dsolve*" handles different types of ODE problems. These include the following:

- Computing closed form solutions for a single ODE ("*dsolve/ODE*", or a system of ODEs ("*dsolve/system*").
- Solving ODEs or a system of them with given initial conditions - boundary value problems ("*dsolve/ICs*").
- Computing formal power series solutions for a linear ODE with polynomial coefficients ("*dsolve/formal_series*").
- Computing formal solution for a linear ODE with polynomial coefficients. ("*dsolve/formal_solution*").
- Computing solutions using integral transforms - Laplace and Fourier ("*dsolve/integral_transform*").
- Computing numerical or series solutions ("*dsolve/series*") for ODEs or systems of ODEs.

A special appliance is the *ODE Analyzer Assistant*, a point-and-click interface to the ODE solver routines. With a lot of choices for interactive solving and plotting differential equations, it can compute numeric and exact solutions and plot the solutions. The basic calling procedures are *dsolve[interactive*] and *worksheet/interactive/dsolve*, and allow changing the simulation parameters in an interactive way [9,10].

2.2.2 Graphical approach

DETools [DePlot] – is an appliance which plots solutions to a system of differential equations. The basic calling sequence is as follows:

DEplot (deqns, vars, trange, options)

Parameters:

deqns - list or set of first order ordinary differential equations, or a single differential equation of any order;
dproc - a Maple procedure representation for first order ordinary differential equations, or a single differential equation of any order;
vars - dependent variable, or list or set of dependent variables;
trange - range of the independent variable;
number - equation of the form 'number'=integer indicating the number of differential equations when deqns is given as a function (dproc) instead of expressions;
options - (optional) equations of the form keyword=value;

Given a set or list of initial conditions, and a system of first order differential equations or a single higher order differential equation, *DEplot* plots solution curves, by numerical methods. This means that the initial conditions of the problem must be given in standard form, that is, the function values and all derivatives up to one less than the differential order of the differential equation at the same point.

3 Results and Discussion

The analysis was made in the following experimental context:

- The SMA spring must develop, at high temperature, the necessary force to lift the barrier arm by S=12 mm

In this context, some reference experimental values were considered [6,7];

- m_1= 0,00642kg, m_2=0,028688kg , k=0,1573373;
- a force F=1.65N in order to get the motion started.

This context produced a value of λ= 0.32.

3.1 The Solution of the Differential Equation

The barrier arm is rising in a very short time, so the dynamical system associated to this phenomenon needs an approach in discrete time, for few units. It was used the "*dsolve*" procedure for getting the solution of the Cauchy problem associated to the model. The solution was asked in series form, taking into account a future aim of getting some approximate evaluations.

In order to simplify the relations, the notation q=x is used. Also, taking into account that at t=0, the controller is at resting, there were considered the following two sets of initial conditions:

(i) $x(0.02) = 0.866803, D(x)(0.02) = 0.897666$
(ii) $x(0.03) = 1.733863, D(x)(0.03) = 1.166965$

Here D(x) denotes the first derivative for x(t).

The solution form for the cases i and ii, is depicted as follows in Figs. 2 and 3.

$$\begin{aligned} ans1 := x(t) &= \frac{866803}{1000000} + \frac{448833}{500000}\left(t - \frac{1}{50}\right) \\ &+ \frac{4}{25}\cos\left(\frac{866803}{1000000}\right)\left(t - \frac{1}{50}\right)^2 \\ &- \frac{149611}{3125000}\sin\left(\frac{866803}{1000000}\right)\left(t - \frac{1}{50}\right)^3 + \Bigg(\\ &- \frac{8}{1875}\cos\left(\frac{866803}{1000000}\right)\sin\left(\frac{866803}{1000000}\right) \\ &- \frac{67150353963}{6250000000000}\cos\left(\frac{866803}{1000000}\right)\Bigg)\left(t - \frac{1}{50}\right)^4 + \Bigg(\\ &- \frac{448833}{195312500}\cos\left(\frac{866803}{1000000}\right)^2 + \frac{149611}{195312500}\sin\left(\frac{866803}{1000000}\right)^2 \\ &+ \frac{30139294820275179}{15625000000000000000}\sin\left(\frac{866803}{1000000}\right)\Bigg)\left(t - \frac{1}{50}\right)^5 \\ &+ \mathrm{O}\left(\left(t - \frac{1}{50}\right)^6\right) \end{aligned}$$

Fig. 2. The solution of the Cauchy problem (8), in series form. Case i of initial conditions

$$\begin{aligned} ans2 := x(t) &= \frac{1733863}{1000000} + \frac{233393}{200000}\left(t - \frac{3}{100}\right) \\ &+ \frac{4}{25}\cos\left(\frac{1733863}{1000000}\right)\left(t - \frac{3}{100}\right)^2 \\ &- \frac{233393}{3750000}\sin\left(\frac{1733863}{1000000}\right)\left(t - \frac{3}{100}\right)^3 + \Bigg(\\ &- \frac{8}{1875}\cos\left(\frac{1733863}{1000000}\right)\sin\left(\frac{1733863}{1000000}\right) \\ &- \frac{54472292449}{3000000000000}\cos\left(\frac{1733863}{1000000}\right)\Bigg)\left(t - \frac{3}{100}\right)^4 + \Bigg(\\ &- \frac{233393}{78125000}\cos\left(\frac{1733863}{1000000}\right)^2 + \frac{233393}{234375000}\sin\left(\frac{1733863}{1000000}\right)^2 \\ &+ \frac{12713451751549457}{3000000000000000000}\sin\left(\frac{1733863}{1000000}\right)\Bigg)\left(t - \frac{3}{100}\right)^5 \\ &+ \mathrm{O}\left(\left(t - \frac{3}{100}\right)^6\right) \end{aligned}$$

Fig. 3. The solution of the Cauchy problem (8), in series form. Case ii of initial conditions

It can be seen that in both simulation cases, the solution has an asymptotic form with the same order 6, and a quite complex expression. In the meantime, it must be noticed the form $t - \frac{1}{50}$ and $t - \frac{3}{100}$ respectively, which appear in the series expressions above.

In order to collect more data about the solution behavior, a computational testing was realized, too.

3.2 Graphical Comparative Analysis of the Solution Behavior

There were used two computational tools for comparing the results: the interactive tool "ODEAnalyzer" and the "DEPlot" appliance for the differential equations.

3.2.1 "ODE Analyzer" results

"ODE Analyzer" has a lot of choices for interactive solving and plotting differential equations. For the present aim we choose the classic Runge-Kutta-Fehlberg 4-5th order method, which has a good accuracy [9]. The interactive assistant shows the value of x(t) and the derivative D(x)(t) at a certain time, together with the plot of the solution.

For a better comparison of the results, there were considered the same two sets i and ii of initial conditions. This matches with taking into account for the controller arm, a displacement of 2mm and 4 mm, corresponding to the action times of 0.02 and 0.03 sec respectively.

Figs. 4 and 5 exhibit the interactive analysis for the study cases i and ii. There are exhibited in the same frame both the plot and a value of x and D(x) at a certain moment. The interest is for the moment t=25.

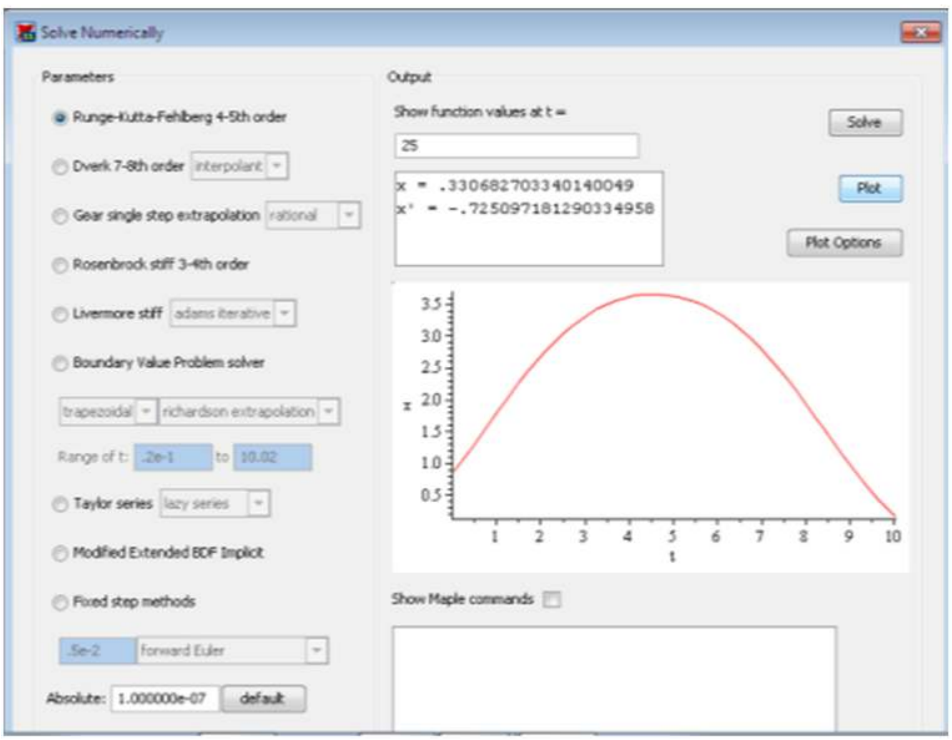

Fig. 4. The case i of analysis for the solution

3.2.2 "DE Plot" results

For this analysis we need the variables domain. Therefore, the variables calculated domain is the following:

$$t = 0..25, x = 0..5.207963$$

The last value for x corresponding to a displacement of 12 mm for the controller arm. The simulations were realized for the above cases i and ii, too. Further, the influence of increasing and decreasing time was taken into account. In the following, the cases are labeled on figures.

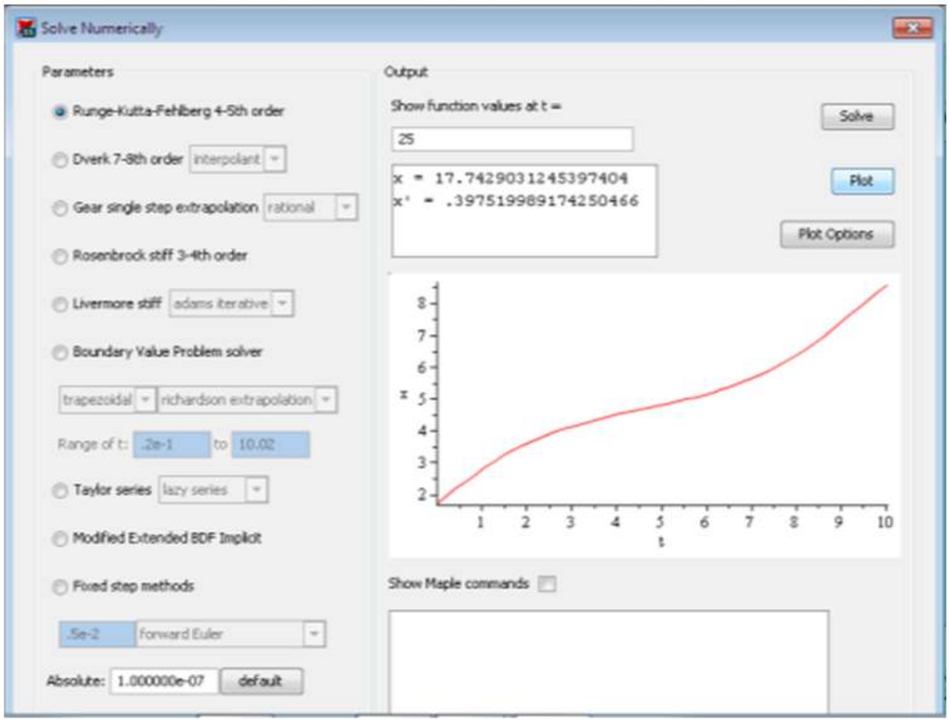

Fig. 5. The case ii of solution analysis

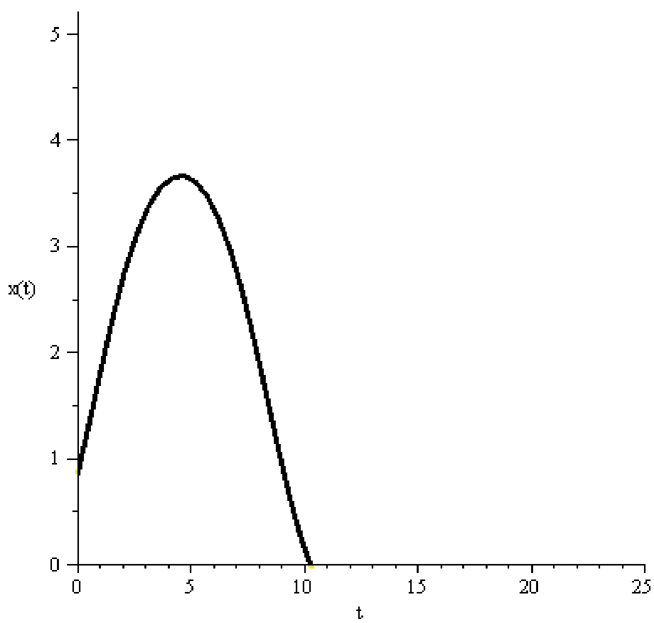

Fig. 6. The case i of initial conditions, t=0..25

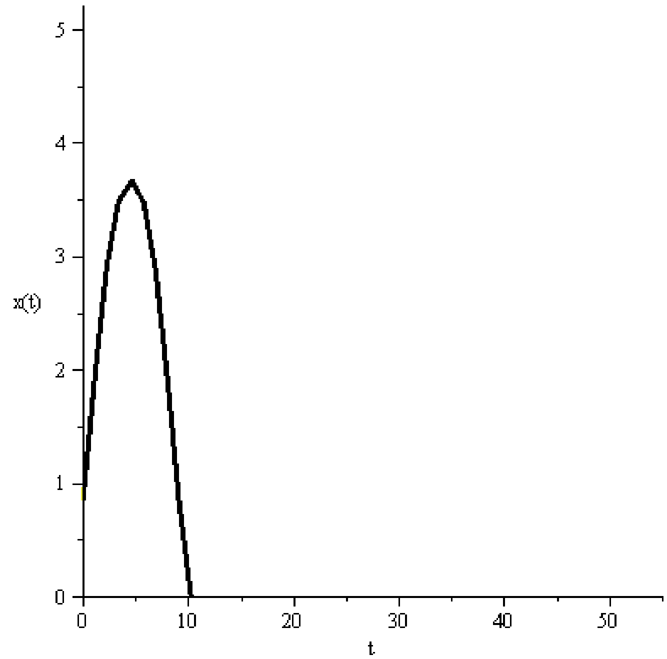

Fig. 7. The case i , t=0..55

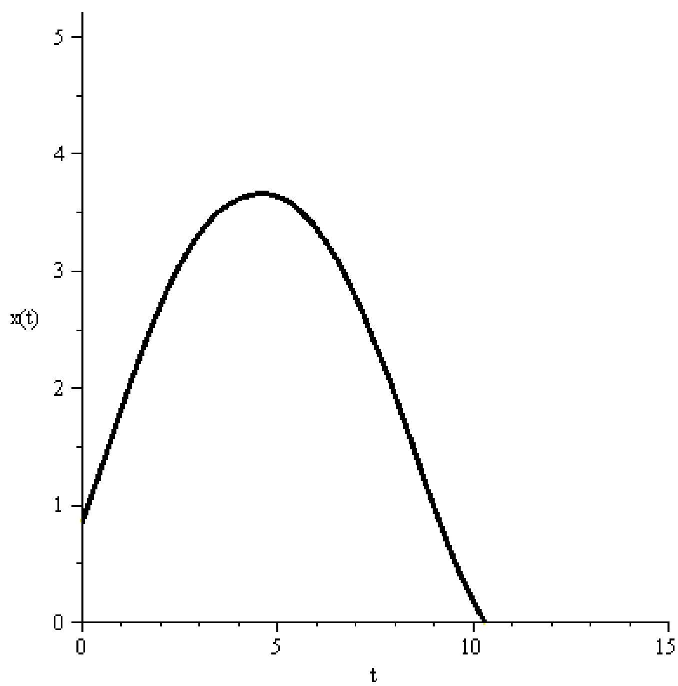

Fig. 8. The case i, t=0..15

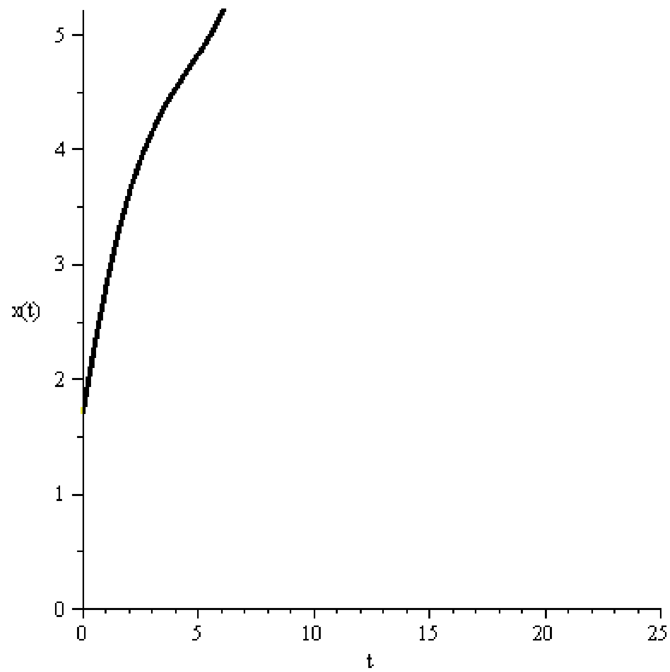

Fig. 9. The case ii of simulation, t=0..25

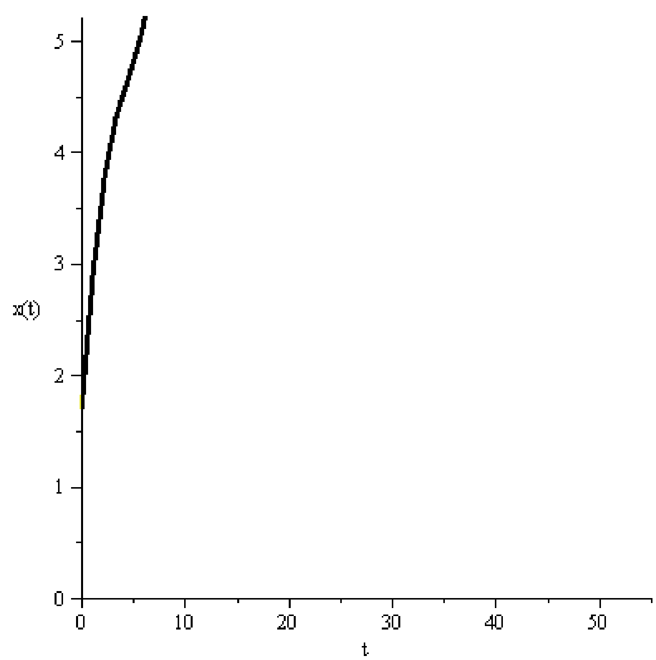

Fig. 10. The case ii, t=0..55

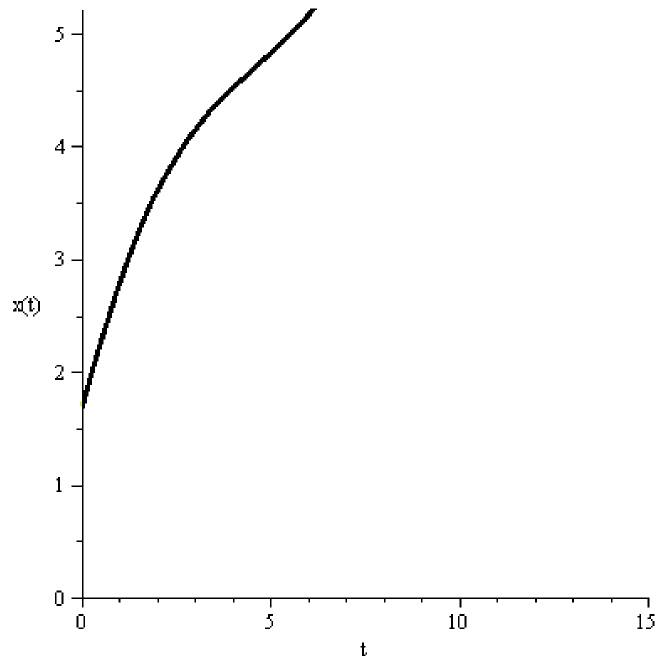

Fig. 11. The case ii, t=0..15

Looking at the above analysis, there issue special remarks.

1. In the first part of analysis, the interactive assistant produced quite different values for the time derivative in the cases i and ii. The negative values for the time derivative indicates that the barrier arm is rising up and then coming down, in the time t=25 units asked in the interactive frame. This is feasible from the experimental design standpoint.
   For all that, a next aim is rising up: to realize a consistent analysis of the solution of the equation (8), taking into account the complicated asymptotic form of it, as presented in Figs. 2 and 3. A closer and refined analysis of this nonlinear differential equation would produce useful data for the model.
2. The solution trajectory changes its allure in both analysis types, in the second case ii, comparing to the first case i. This allows us to consider this model a *model sensitive to initial conditions*, and the fact is matched from experimental standpoint, since the phenomena in the spring acts in a very short time, and therefore is not obvious to control it. From the plots it seems that the maximum displacement could be reached until 10 sec, and this is reliable from the experimental design standpoint.
3. The trajectory trend for larger time units provide that after a small time the arm restarts its activity. Is the case of Fig. 7, corresponding to the first parameter case – when the trajectory prepares to start again a cycle, and Fig. 10, corresponding to the second parameter case – when the trend is relaxed. This fact confirms the periodicity of the barrier activity and allows the further study of initial value problem [10].
4. The *DEplot* appliance is good in giving information about the influence of time units on the trajectory behavior, and this is helping the experimental design standpoint. In Maple we can realize simulations with the time units as large as needed [11]. On the other hand, *ODEAnalyser* allows refining the parameters as needed. Therefore, another next aim is to test the dynamic system for another λ, from computational standpoint [12], and to take into account the results in future experiment designs. The further data would provide information about future analytical issues, one of which is the following: could be λ an invariant for the equation (8)?

5. The action time is very important in the experiment [13]. This fact is exhibited also in the above computational analysis, referring to the modification of the trajectory trend – “steep” and “relaxed” respectively, in the plots. Our model is validated in a particular case given by λ= 0.32 and the time t=0..25, matched from experimentally design standpoint. From the pictures it can be seen that in the proposed conditions the barrier arm realizes a complete moving, rising up and back down. In the same conditions, the solution of the model can be analytically deduced, as presented in above section 3.1, and that completes the validation.

4 Conclusion

In this paper it is realized an analytical evaluation and computational testing for the mathematical model associated to the access control structure with SMA spring. The conclusion has two parts: on one hand, the solution of the model is hard to approach by hand. Therefore it is necessary to refine the analytical methods for approach. On the other hand, the computational approach exhibited a model sensitive to initial conditions. Therefore, further tests of the model are necessary, taking into account the influence of all SMA spring parameters. A next aim is to consider the relationship between the temperature and the electric intensity in the spring, and their influence on the analytical solution of the model.

Competing Interests

Authors have declared that no competing interests exist.

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