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A 2-Step Four-Point Hybrid Linear Multistep Method for Solving Second Order Ordinary Differential Equations Using Taylor's Series Approach

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Abstract

This paper considers the development of a 2-step four-point continuous hybrid method for the direct solution of initial value problem (IVPs) of second order ordinary differential equations using the method of interpolation of the power series approximate solution and collocation of the differential system to develop our scheme. Taylor's series approximation is used to analyze and implement y_{n+i} $i=1\ldots n-1$ at $x_{n+i}, \mathrm{j}=0(1)2.$ The method is found to be consistent and zero-stable. Numerical results show a superior accuracy compared to existing methods.

Keywords: Second order initial value problems; power series; interpolation; collocation; Taylor's series, efficiency.

1 Introduction

The focus of this paper is based on initial value problems (IVPs) of general second order ordinary differential equations (ODEs) of the form:

$$y'' = f(x, y, y'), y(a) = y_0, y'(a) = \beta \tag{1}$$

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where f is a given real valued function which is continuous within the interval of integration. We assumed that f satisfied the Lipstich condition that guaranted the existence and the uniqueness of the solution to equation (1). Many of such problems in (1) may not easily be solved analytically, hence there is need to develop numerical schemes to solve (1) directly. Equation (1) is often reduced to system of first order ordinary differential equations in which Numerical method for first order odes are used to solved them. Linear multistep method has been proposed to solve equaion (1) without reducing it to systems of first order ordinary differential equation, they include Awoyemi [1] and [2], Awoyemi and Kayode [3], Adesanya et al. [4], Badmus and Yahaya [5]. Lambert [6], also discussed an optimal two step method called the numerov's method. Awoyemi [1] particularly proposed a two-step hybrid multistep method with continuous coefficients for the solution of (1) and Kayode [7] proposed a three-step one point hybrid method based on collocation at selected grid and off-grid points. All the individuals proposed methods were implemented in predictor-correct and block method respectively and adopted Taylor series expansion to supply starting value. This method retaining certain characteristics of the continuous linear multistep method share with Runge-Kutta methods the property of utilizing data at other points, thod obtained by various authors retained certain charateristics of the continuous linear multistep method shared with Runge-Kutta method the property of utilizing data at other points, other than the step point. $x_{n+j}, j = 0,1\cdots n-1$. This is useful in reducing the step number of a method and still remain zero stable. In this research, two–step, two point hybrid linear multistep method based on collocation at selected grid and off-grid point was developed for solving second order initial value problems of ordinary differential equation using hybrid method $x_{n+j,} j = 0,1\cdots n-1$ with Taylor's series been used as the starting value and for implementation.

2 The Derivation of the Method

In this section, we apply the interpolation and collocation procedures and we chose our interpolation at the grid points (i) and our collocation points (c) at both grid and off-grids points. We consider a power series in the form:

$$y(x) = \sum_{j=0}^{(i+c)-1} a_j x^j \tag{2}$$

The first and second derivatives are

$$y'(x) = \sum_{j=1}^{(i+c)-1} j a_j x^{j-1} \tag{3}$$

$$y'' = \sum_{j=2}^{(i+c)} j\left(j-1\right) a_j x^{j-2} \tag{4}$$

Combining (3) and (4) generates the differential system

$$y'' = \sum_{j=2}^{(i+c)-1} j(j-1) a_j x^{j-2} = f(x, y, y') \tag{5}$$

Collocating (5) at $x = x_{n+i}, i = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2$ and interpolating (2) at $x = x_{n+i}, i = 1, \tfrac{3}{2}$ result into a system of equation

$$\sum_{j=2}^{(i+c)-1} j(j-1)a_j x^{j-2} = f_{n+i} \quad i = \left[0, \frac{1}{2}, 1, \frac{3}{2}, 2\right] \tag{6}$$

and

$$\sum_{j=0}^{(i+c)-1} a_j x^j = y_{n+i} \quad i = \left[1, \frac{3}{2}\right] \tag{7}$$

where

$$x_{n+i} = x_n + ih$$

As stated above, the collocation and interpolation process results into the equation stated below:

$$\begin{aligned}
f_n &= 2a_2 + 6a_3x_n + 12a_4x_n^2 + 20a_5x_n^3 + 30a_6^4 \\
f_{n+1/2} &= 2a_2 + 6a_3x_{n+1/2} + 12a_4x_{n+1/2}^2 + 20a_5x_{n+1/2}^3 + 30a_6x_{n+1/2}^4 \\
f_{n+1} &= 2a_2 + 6a_3x_{n+1} + 12a_4x_{n+1}^2 + 20a_5x_{n+1}^3 + 30a_6x_{n+1}^4 \\
f_{n+3/2} &= 2a_2 + 6a_3x_{n+3/2} + 12a_4x_{n+3/2}^2 + 20a_5x_{n+3/2}^3 + 30a_6x_{n+3/2}^4 \\
f_{n+2} &= 2a_2 + 6a_3x_{n+2} + 12a_4x_{n+2}^2 + 20a_5x_{n+2}^3 + 30a_6x_{n+2}^4 \\
y_{n+1} &= a_0 + a_1x_{n+1} + a_2x_{n+1}^2 + a_3x_{n+1}^3 + a_4x_{n+1}^4 + a_5x_{n+1}^5 + a_6x_{n+1}^6 \\
y_{n+3/2} &= a_0 + a_1x_{n+3/2} + a_2x_{n+3/2}^2 + a_3x_{n+3/2}^3 + a_4x_{n+3/2}^4 + a_5x_{n+3/2}^5 + a_6x_{n+3/2}^6
\end{aligned} \tag{8}$$

Applying gaussian elimination method to the systems of equation (8), yields the value of $a's$ as follows:

$$a_6 = \frac{h^4}{45}f_{n+2} - \frac{4h^4}{45}f_{n+3/2} + \frac{2h^4}{15}f_{n+1} - \frac{4h^4}{45}f_{n+}\,{}^1\!/_2 + \frac{h^4}{45}f_n$$

$$a_5 = -\frac{h^4}{30}(3h+4x_n)f_{n+2} + \frac{h^4}{15}(7h+8x_n)f_{n+3/2} - \frac{4h^4}{5}(x_n+h)f_{n+1} + \frac{h^4}{15}(9h+8x_n)f_{n+1/2} - \frac{h^4}{30}(5h+4x_n)f_n$$

$$\begin{aligned}
a_4 = {} & \frac{h^4}{72}\left(11h^2 + 36x_nh + 24x_n\right)f_{n+2} - \frac{h^4}{9}\left(7h^2 + 21x_nh + 12x_n^2\right)f_{n+3/2} \\
& + \frac{h^4}{12}\left(19h^2 + 48x_nh + 24x_n^2\right)f_{n+1} - \frac{h^4}{9}\left(13h^2 + 27x_nh + 12x_n^2\right)f_{n+1/2} \\
& + \frac{h^4}{72}\left(35h^2 + 60x_nh + 24x_n^2\right)f_n
\end{aligned}$$

$$a_3 = -\frac{h^4}{36}\left(3h^3 + 22x_n h^2 + 36x_n^2 h + 16x_n^3\right) f_{n+2} + \frac{2h^4}{9}\left(2h^3 + 14x_n h^2 + 21x_n^2 h + 8x_n^3\right) f_{n3/2}$$
$$-\frac{h^4}{3}(3h^3 + 19x_n h^2 + 24x_n^2 h + 8x_n^3) f_{n+1} + \frac{2h^4}{9}\left(6h^3 + 26x_n h^2 + 27x_n^2 h + 8x_n^3\right) f_{n+1/2}$$
$$-\frac{h^4}{36}\left(25h^3 + 70x_n h^2 + 60x_n h + 16x_n^3\right)$$

$$a_2 = \frac{h^4}{12x_n}\left(3h^3 + 11x_n h^2 + 12x_n^2 h + 4x_n^3\right) f_{n+2} - \frac{2h^4}{3x_n}\left(2h^2 + 7x_n h^2 + 7x_n^2 h + 2x_n^3\right) f_{n+3/2}$$
$$+\frac{h^4}{2x_n}(6h^3 + 19x_n h^2 + 16x_n^2 h + 4x_n^3) f_{n+1} - \frac{2h^4}{3x_n}(6h^3 + 13x_n h^2 + 9x_n^2 h + 2x_n^3) f_{n+1/2}$$
$$+\frac{h^4}{12}\left(6h^4 + 25x_n h^3 + 35x_n^2 h^2 + 20x_n^3 h + 4x_n^4\right) f_n$$

$$a_1 = \frac{2}{h} y_{n+3/2} - \frac{2}{h} y_{n+1}$$
$$+\frac{h^4}{2880}\left(33h^5 - 720x_n^2 h^3 - 1760x_n^3 h^2 - 1440x_n^4 - 384x_n^5\right) f_{n+2}$$
$$-\frac{h^4}{720}\left(71h^5 - 960x_n^2 h^3 - 2240x_n^3 h^2 - 1680x_n^4 h - 384x_n^5\right) f_{n+3/2} \qquad (9)$$
$$-\frac{h^4}{480}\left(161h^5 + 1440x_n^2 h^3 + 3040x_n^3 h^2 + 1920x_n^4 h + 384x_n^5\right) f_{n+1}$$
$$-\frac{h^4}{720}\left(477h^5 + 2880x_n^2 h^3 + 4160x_n^3 h^2 - 2160x_n^4 h - 384x_n^5\right) f_{n+1/2}$$
$$-\frac{h^4}{2880}\left(475h^5 + 2880x_n h^4 + 6000x_n^2 h^3 + 5600x_n^3 h^2 + 2400x_n^4 h + 384x_n^5\right) f_n$$

$$a_0 = y_n - a_1(x) - a_2(x)^2 - a_3(x)^3 - a_4(x)^4 + a_5(x)^5 + a_6(x)^6$$

Substituting the values of $a_i \;\; i = 0,1\cdots 6$ into (2) results into equation of the form:

$$y(x) = y_n + \sum_{i=1}^{6} a_i \left(x^i - x_n^i\right) \qquad (10)$$

Now using the transformation:

$$t = \frac{x - x_n}{h}$$

$\dfrac{dt}{dx} = \dfrac{1}{h}$ (10) becomes:

$$y_{n+2} - 2t\left(y_{n+3/2}\right) + (2t-1)\, y_{n+1} = \frac{h^2}{2880}\begin{bmatrix} \left(-11t+80t^3-40t^4-96t^5+64t^6\right) f_n + \left(19t-160t^3+169t^4+48t^5-64t^6\right) f_{n+1/2} \\ \left(-97t+248t^2+200t^4+91t^6\right) f_{n+1} + \left(-55t+648t^3+231t^4-48t^5-64t^6\right) f_{n+3/2} + \\ \left(17t-80t^3-40t^4+96t^5+64t^6\right) f_{n+2} \end{bmatrix} \tag{11}$$

The coefficients are put as follows:

$$\begin{aligned} \alpha_1 &= -2t+1 = (1-2t) \\ \alpha_{3/2} &= 2t \\ \beta_0 &= \frac{h^2}{2880}\left(-11t+80t^3-40t^4-96t^5+64t^6\right) \\ \beta_{1/2} &= \frac{h^2}{720}\left(19t-160t^3+160t^4+48t^5-64t^6\right) \\ \beta_1 &= \frac{h^2}{480}\left(-97t+240t^2-200t^4+64t^6\right) \\ \beta'_{3/2} &= \frac{h^2}{720}\left(-55t+160t^3+160t^4-48t^5-64t^6\right) \\ \beta_2 &= \frac{h^2}{2880}\left(17t-80t^3-40t^4+96t^5+64t^6\right) \end{aligned} \tag{12}$$

The first derivatives of (12) are follows:

$$\begin{aligned} \alpha'_1 &= -\frac{2}{h} \\ \alpha'_{3/2} &= \frac{2}{h} \\ \beta'_0 &= \frac{h}{2880}\left(-11+240t^2-160t^3-480t^4+384t^5\right) \\ \beta'_{1/2} &= \frac{h}{720}\left(19-480t^2+640t^3+240t^4-384t^5\right) \\ \beta'_1 &= \frac{h}{480}\left(-97+480t-800t^3+384t^5\right) \\ \beta'_{3/2} &= \frac{h}{720}\left(-55+480t^2+640t^3-240t^4-384t^5\right) \\ \beta'_2 &= \frac{h}{2880}\left(17-240t^2-160t^3+480t^4+384t^5\right) \end{aligned} \tag{13}$$

Evaluating equation (12) at $t=1$ which implies that $x = x_{n+2}$ gives:

$$y_{n+2} = 2y_{n+3/2} - y_{n+1} + \frac{h^2}{960}\left(19f_{n+2} + 204f_{n+3/2} + 14f_{n+1} + 4f_{n+1/2} - f_n\right) \tag{14}$$

With the order of accuracy 5 and Error constant $c_7 = -\frac{163}{5000000}$

The first derivative is given by evaluating (13) at $x = x_{n+2}$.

$$y'_{n+2} = \frac{1}{h}\left(-2y_{n+1} + 2y_{n+3/2}\right) + \frac{h}{2880}\left(481f_{n+2} + 1764f_{n+3/2} - 198f_{n+1} + 140f_{n+1/2} - 27f_n\right) \quad (15)$$

where

$$f_{n+i} = f\left(x_{n+i}, y_{n+i}, y'_{n+i}\right), i = 0, 1/2, 1, 3/2, 2, \quad (16)$$

3 Taylor's Series Expansion of the Method

Taylor's series expansion is also used to calculate the value of y_{n+i} and y'_{n+i}, their first and second derivatives at $x = x_n$ in equation (16) are as follows:

$$y_{n+i} = y\left(x_n + ih\right) = y_n + ihy'\left(x_n\right) + \frac{(ih)^2}{2!} f_n + \cdots$$

$$y'_{n+i} = y'\left(x_n\right) + ihf_n + \frac{(ih)^2}{2!} f'_n + \cdots$$

$$f_{n+i} = y''\left(x_n + ih\right)$$

$$y''\left(x_n + ih\right) = f_n + ihf'_n + \frac{(ih)^2}{2!} f''_n + \frac{(ih)^3}{3!} f'''_n + \frac{(ih)^4}{4!} f_n^{iv} + \frac{(ih)^5}{5!} f_n^{v} + \cdots$$

where

$$f_n = f\left(x_n, y_n, y'_n\right)$$
$$f_n^{(i)} = f^{(i)}\left(x_n, y_n, y'_n\right), i = 1, 2, 3 \cdots$$

In addition, we write equation (16) in the form

$$f = f\left(x, y, y'\right) \quad (17)$$

We find f', f'' and f''' by the use of partial derivatives techniques as stated below:

$$f' = \frac{df}{dx} = \frac{\partial f}{\partial x} + y'_n \frac{\partial f}{\partial y} + f \frac{\partial f}{\partial y'}$$

$$f'' = \frac{d^2 f}{dx^2} = 2\left(Ay' + Bf\right) + Cfy' + D + E$$

$$f''' = \frac{d^3 f}{dx^3} = 2G + 3\left(Hy' + If\right) + Jfy' + K + L + M$$

where

$$A = \frac{\partial^2 f}{\partial x \partial y} + f\,\frac{\partial^2 f}{\partial y \partial y'}$$

$$B = \frac{\partial^2 f}{\partial x \partial y'}$$

$$C = \frac{\partial f}{\partial x} + y'\,\frac{\partial f}{\partial y} + f\,\frac{\partial f}{\partial y'}$$

$$D = \frac{\partial^2 f}{\partial x^2} + \left(y'\right)^2 \frac{\partial^2 f}{\partial y^2} + f^2\,\frac{\partial^2 f}{\left(\partial y'\right)^2}$$

$$E = f\,\frac{\partial y}{\partial y}$$

$$G = y'f'\,\frac{\partial^2 f}{\partial y \partial y'} + f'\,\frac{\partial^2 f}{\left(\partial y'\right)^2} + y'ffy'\,\frac{\partial^2 f}{\partial y \partial y'} + f'\,\frac{\partial^2 f}{\partial x \partial y}$$

$$H = \frac{\partial^3 f}{\partial x \partial y} + y'\,\frac{\partial^3 f}{\partial x \partial y^2} + f\,\frac{\partial^2 f}{\partial y^2} + y'f\,\frac{\partial^3 f}{\partial y^2 \partial y'} + f^2\,\frac{\partial^3 f}{\partial y\left(\partial y'\right)^2} + 2\,\frac{\partial^3 f}{\partial x \partial y \partial y'}$$

$$I = \frac{\partial^3 f}{\partial x^2 \partial y'} + \frac{\partial^2 f}{\partial x \partial y} + f\,\frac{\partial^3 f}{\partial y\left(\partial y'\right)^2} + f\,\frac{\partial^2 f}{\partial y \partial y'}$$

$$J = f\,\frac{\partial f}{\partial y} + \partial y'\,\frac{\partial^2 f}{\partial x \partial y} + f\,\frac{\partial^3 f}{\partial y\left(\partial y'\right)^2} + f\,\frac{\partial^2 f}{\partial y \partial y'}$$

$$K = \left(\frac{\partial f}{\partial x} + y'\,\frac{\partial f}{\partial y} + f\,\frac{\partial f}{\partial y'}\right)\left[\frac{\partial^2 f}{\partial x \partial y'} + y'\,\frac{\partial^2 f}{\partial y \partial y'} + f\,\frac{\partial^2 f}{\left(\partial y'\right)}\right]$$

$$L = \frac{\partial^3 f}{\partial x^3} + f^3\,\frac{\partial^3 f}{\left(\partial y'\right)^3} + \left(y'\right)^3 \frac{\partial^3 f}{\partial y^3}$$

$$M = f'\,\frac{\partial f}{\partial y}$$

4 Analysis of the Basic Properties of the Method

4.1 Order and Error Constant of the Method

In finding the order, we adopt the method proposed by Lambert [5], with the linear operator:

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h^2 \sum_{j=0}^{k} \beta_j f_{n+j} \tag{18}$$

We associate the linear operator L with the 2-steps scheme and define as

$$L\left\{y(x), h\right\} = \sum_{j=0}^{k} \left[\alpha_j y(x+jh) - h^2 \beta_j y''(x+jh)\right] \tag{19}$$

Where α_0 and β_0 are both non-zero and $y(x)$ is an arbitrary function which is continuous and differentiable on the interval [a, b]. If we assume that $y(x)$ has as many higher derivatives as we require, then on Taylor's series expansion about the point x, we obtain

$$L\left[y(x,h)\right] = c_0 y(x) + c_1 h y'(x) + \cdots c q h^q y^q(x) + \cdots \tag{20}$$

Accordingly we say that the method has order P if,

$$c_0 = c_1 = \cdots = c_p = c_{p+1} = 0, c_{p+2} \neq 0$$

Then, c_{p+2} is the error constant and $C_{p+2} h^{p+2} y^{p+2}(x_n)$ is the principal local truncation error at the point x_n

Expanding (19) and comparing coefficients, we get the following c_i values for the method

$$y_{n+2} = 2y_{n+3/2} - y_{n+1} + \frac{h^2}{960}\left(19 f_{n+2} + 204 f_{n+3/2} + 14 f_{n+1} + 4 f_{n+1/2} - f_n\right)$$

$$c_0 = 1 - 2 + 1 = 0$$

$$c_1 = 2 + 1 - 3 = 0$$

$$c_2 = 2 - \frac{9}{4} + \frac{1}{2} - \frac{19}{960} - \frac{204}{960} - \frac{14}{960} - \frac{4}{960} + \frac{1}{960} = 0$$

$$c_3 = \frac{8}{6} - \frac{27}{24} + \frac{1}{6} - \frac{38}{960} - \frac{306}{960} - \frac{14}{960} - \frac{2}{960} = 0$$

$$c_4 = \frac{16}{24} - \frac{81}{192} + \frac{1}{24} - \frac{38}{960} - \frac{459}{1920} - \frac{7}{960} - \frac{1}{1920} = 0$$

$$c_5 = \frac{32}{120} - \frac{243}{1920} + \frac{1}{120} - \frac{76}{2880} - \frac{459}{3840} - \frac{7}{2880} - \frac{1}{11520} = 0$$

$$c_6 = \frac{64}{720} - \frac{729}{23640} + \frac{1}{720} - \frac{38}{2880} - \frac{1377}{36720} - \frac{7}{11520} - \frac{1}{92160} = 0$$

$$c_7 = \frac{128}{5040} - \frac{243}{35840} + \frac{1}{5040} - \frac{19}{3600} - \frac{41311}{307200} - \frac{7}{57600} - \frac{1}{921600} = -\frac{163}{5000000}$$

We find that $c_0 = c_1 = c_2 = \cdots = c_6$ and $c_7 = c_{p+2} \neq 0$ which implies that the method is of order 5 and error constant $c_{p+2} = -\frac{163}{5000000}$.

5 Consistency of the Method

For this method to be consistent, the following criteria must be satisfied

Condition 1: $p \geq 1$

The order is 5. Therefore, condition (1) is satisfied.

Condition 2: $\sum_{j=0}^{k} \alpha_j = 0$ where $j = (0 \cdots 2)$

$$\alpha_0 + \alpha_{1/2} + \alpha_1 + \alpha_{3/2} + \alpha_2 = 0$$

$$0 + 0 + 1 - 2 + 1 = 0$$

This also satisfy condition (2)

Condition 3: $\rho'(r) = 0$ when $r = 1$

$$\rho = r^2 - 2r^{3/2} + r$$

$$\rho' = 2r - 3r^{1/2} + 1 = 0 \text{ when } r = 1$$

Condition 4: $\rho''(r) = 2!\sigma(r)$ when $r = 1$

$$\rho'' = 2 - \frac{3}{2}r^{-1/2} \quad r = 1, \quad \rho'' = \frac{1}{2}$$

$$\sigma(r) = \frac{1}{960}\left(19r^2 + 204r^{3/2} + 14r + 4r^{1/2} - 1\right)$$

$$\sigma(1) = \frac{1}{4}$$

and

$$2!\sigma(1) = 2\left(\frac{1}{4}\right) = \frac{1}{2}$$

Since $\rho''(r) = 2!\sigma(r)$ at $r = 1,$ then the condition (4) is satisfied.

Thus, this method is said to be consistent.

6 Zero Stability

Definition [5]: A linear multistep method is said to be zero-stable, if no root of the first characteristics polynomials $\rho(r)$ has modulus greater than one and if every root of modulus one has multiplicity not greater than two.

The scheme is zero stable when no root of the first characteristics polynomial $\rho\left(r\right)$ has modulus greater than one that is $\left|r\right| \leq 1$

A method is zero stable if $\sum_{j=0}^{k} \alpha_j = 0$, where α_j are the coefficients of $\sum_{j=0}^{k} \alpha_j y_{n+j}$

$$\sum_{j=0}^{k} \alpha_j = \alpha_0 + \alpha_{1/2} + \alpha_1 + \alpha_{3/2} + \alpha_2 = 0 + 0 + 1 - 2 + 1 = 0$$

Thus, this method is said to be zero stable.

7 Implementation of the Method

Two test problems were solved using the mesh size of $h = 1/320$ and $h = 0.1$ for $n = 10$, which automatically changes with different n to test the efficiency of this method. The errors arising from both problems were compared with the errors from different method.

8 Numerical Problems

Problem 1

$$y'' + y = 0, \qquad y(0) = 1 = y'(0),$$

$$h = 0.1$$

Analytical Solution

$$y(x) = \cos x + \sin x$$

Problem 2

$$y'' = \frac{(y')^2}{2y} - 2y, y\left(\frac{\pi}{6}\right) = \frac{1}{4}, y'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},$$

$$h = \frac{1}{320}$$

Theoretical solution

$$y(x) = Sin^2 x$$

Problem 3

$$y'' = x(y')^2 \quad y(0) = 1,\ y'(0) = \frac{1}{2},\ h = \frac{1}{320}$$

Theoretical solution

$$y(x) = 1 + \frac{1}{2}\log\left(\frac{2+x}{2-x}\right)$$

8.1 Numerical Solutions to Problems 1-3 as Shown in the Tables 1-3

The computational errors of our method tested on problems 1 – 3 compared to other researchers. Problem 1 was compared with Ehigie et al. [7]. Problem 2 was compared with Awoyemi [3], Awoyemi and Kayode [8] while the result of problem 3 was compared with Kayode and Adeyeye [6].

Table 1. Table 1 for problem 1

X	Error in new method	Error in Ehigie et al. [7]
0.1	1.07E-008	4.25E-06
0.2	3.08E-008	8.46E-06
0.3	5.18E-008	1.26E-05
0.4	8.23E-008	1.66E-05
0.5	1.22E-007	2.05E-05
0.6	1.72E-007	2.41E-05
0.7	2.32E-007	2.75E-05
0.8	3.01E-007	3.07E-05
0.9	3.79E-007	3.35E-05
1.0	4.65E-007	3.60E-05

Table 2. Table 2 for problem 2

X	Error in Awoyemi and Kayode [8]	Error in Awoyemi [3]	Error in our method
0.1	6.6391E-14	2.6074E-10	3.33E-016
0.2	2.0012E-10	1.9816E-09	7.77E-016
0.3	1.72007E-09	6.5074E-09	1.11E-015
0.4	5.8946E-08	1.5592E-08	1.66E-015
0.5	1.4434E-08	3.1504E-08	2.27E-015
0.6	4.1866E-08	5.6374E-08	3.86E-014
0.7	5.3109E-08	9.6164E-08	1.89E-013
0.8	9.1131E-08	1.5686E-07	4.19E-013
0.9	1.4924E-07	2.4869E-07	6.74E-013
1.0	2.3718E-07	3.8798E-07	8.87E-013

Table 3. Table 3 for problem 3

X	Error in new method	Error in Kayode and Adeyeye [6]
0.1	1.0284E-011	4.831380E-011
0.2	7.6971E-011	3.382836E-009
0.3	2.6111E-010	1.580320E-008
0.4	6.5442E-010	4.333951E-008
0.5	1.3893E-009	9.391426E-008

9 Conclusions

In this work, we have derived, analysed and implemented a 4-point-2-step hybrid method for the solution of general second order differential equations by adopting power series as the basis function. Collocation and interpolation approach is adopted for the derivation of the method while Taylor's series approach is adopted for its implementation. In Table 1, our method perform better than the method of Ehigie et al. [8] despite the large step number of the scheme, likewise Table 2; showed better accuracy than Awoyemi and Kayode [3] and Awoyemi [2]. Also, despite the implementation of Kayode and Adeyeye [9] our method still performed better than both whom earlier solved the same problems respectively. Thus, the method developed in this paper is efficient and compare favourably with other methods.

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Authors' Contributions

This work was carried out in collaboration between all authors. Author DOA serve as a supervisor for the post graduate degree programme for both authors OOO and OBA. Author OOO post graduate research was based on hybrid linear multistep methods of second order ordinary differential equations using Taylor's series for implementation which form the basis for this paper. Author OBA also worked on one-step Hybrid Linear Multistep Methods which gives the fore-knowledge for this paper. All authors read and approved the final manuscript.

Competing Interests

Authors have declared that no competing interests exist.

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