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Theoretical Approach in Determining Vibrations of Periodic Cutting Tool Holder

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Research Article

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ABSTRACT

Vibration is one of the most annoying problems faced during metal cutting operation, and it occurs frequently in manufacturing industries. The vibration level depends on many different parameters such as material type, rigidity of tooling structure, cutting data and operation mode. In milling the cutting process subjected to the tool vibrations having a milling tool holder will most likely result in high vibration levels. These vibrations have a consequence of reduced tool life, poor surface finish and sound distributions. This study presents a new approach of localization for an elastic periodic cutting tool holder of milling machine. A numerical model has been developed to describe the structure of the cutting tool holder. On the other hand, the behavior of periodic holder is investigated numerically. This paper examined the dominating milling vibration components and identified these vibrations which are related to structural dynamic properties of the milling tool holder.

Keywords: Milling; vibration; modelling; periodic holder.

NOMENCLATURE

A: *cross-sectional area,* m^2; $\mathbf{A_d}$**:** *axial depth of cut, mm;* **b:** *chip width, mm;* **C:** *cutting force coefficient;* **E:** *Young's modulus,* N/m^2; **EI:** *holder rigidity:* **F:** *total force magnitude, N;* $\mathbf{F_u}$**:** *axial force, N;* $\mathbf{F_w}$**:** *bending force, N:* **f:** *transverse force, N/m:* $\mathbf{f_T}$**:** *feed per tooth, mm/tooth:* **h:** *undeformed chip thickness, mm;* **I:** *second moment of area,* m^4; **K:** *coefficients of stiffness matrix;* **KE:** *kinetic energy,* kgm^2/s^2; **l:** *element length, m;* **m:** *equivalent mass per unit length, kg/m;* **M:** *coefficients moment of inertia, Nm;* **N:** *spindle speed, rev/s;* **PE:** *potential energy,* kgm^2/s^2; $\mathbf{R_d}$**:** *radial depth of cut (mm);* **t:** *time, s;* **x:** *axial co-ordinate of the beam holder, m;* **v:** *transverse displacement of the beam, m;* **w:** i^{th} *natural frequency of the beam, rad/s;* **Z:**

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number of cutting edges; ***θ:*** *cutting force angle;* ***ρ:*** *material mass density, kg/m^3;* ***γ:*** *torsional constant.*

1. INTRODUCTION

The basic idea underlying the whole concept of periodic structures is that when a wave is travelling in a medium and meets a transition in those medium characteristics, a part of it will propagate through the new medium and another part will reflect. While, in a regular structure, the wave is expected to travel without any change until it reaches the boundaries of the structure. The ability of periodic structures to transmit waves from one location to another within the pass bands can be greatly reduced when the ideal periodicity is disrupted or disordered. In case of passive structures, the aperiodicity can result from unintentional material, geometric and manufacturing variability (Baz, 2001).

A part of the reflected wave will interact with the incident wave in a manner that will characterize the interference. When constructive interference occurs, the frequency is characterized by being the pass band of the structure; while, in the case of destructive interference, the frequency is characterized by being the stop band of the structure, Gupta (1970). If the structure setup is repeated for several times, it is known as a periodic structure. The destructive effects will show more significantly when the repetitions of the structure unit increase in number, because as the part of the wave that propagates incorporates other similar changes in the medium, another part of it is destructed and so on.

In a reviewing of the research performed in the area of wave propagation in periodic structures (Mead, 1996) defined a periodic structure as a structure that consists fundamentally of a number of identical structural components that are joined together to form a continuous structure. Examples of periodic structures can be seen in fuselages of aircraft, petroleum pipe-lines, railway tracks, and many others. An illustration of a simple periodic tool holder system is shown in figure 1.

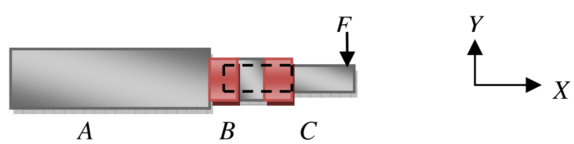


Fig.1. A simple schematic drawing of cutting tool system (A) Spindle, (B) periodic tool holder, and (C) cutting tool

In general, when a wave propagating in a structure encounters a change in the geometry and/or the material properties, the wave is split into two components; a propagating component and a reflected component. The reflected part interacts with the propagating part in a manner that is controlled by the phase difference between them. Studies of the characteristics of one-dimensional periodic structures have been extensively reported by Unger (1966). These structures are easy to analyze due to its geometrical simplicity. Gupta (1970) presented an analysis for periodically-supported beams that introduced the concepts of the cell and the associated transfer matrix. Faulkner and Hong (1985) presented a study of mono-coupled periodic systems. Their study analyzed the free vibration of the spring-mass systems as well as point-supported beams using analytical and finite element methods. Mead and Yaman (1991) studied the response of one-dimensional periodic structures subjected to periodic loading. Their study involved the generalization of the support condition to involve rotational and displacement springs as well as impedances. The effects of the excitation point as well as the elastic support characteristics on the pass and stop characteristics of the beam are presented. Later, Mead (1994) proved that the power transmission in both direction of a simply supported beam excited by a point force was equal regardless of the excitation location. Those results were generalized by Langley (1996) to prove the same for generalized supports in the absence of damping.

The vibration of cutting tool system under certain conditions has long been recognized as one of the most significant factors affecting the performance of a machine tool. In the past, several methods for the identification of milling vibrations have been proposed. Tlusty and Zaton (1983) considered the use of stability chart to predict self-excited vibration. It has been shown that, the complex mathematical calculations for milling dynamics based on large amount of cutting forces data are required to predict the onset of vibration by using the stability charts. Jalili Saffar (2008) proposed a simulation to predict cutting forces and tool deflection during end milling operation, and to verify the accuracy of simulation results compared with those based on the theoretical relationships.

2. TRANSFER MATRIX ANALYSIS

The transfer matrix approach, in general, is based on developing a relation between two ends of the structure element. The real power of the transfer matrix approach comes when the structure can be divided into a set of substructures with a set of elements and nodes that are connected to another set on some fictitious boundary inside the structure. Using the method of static condensation, the internal nodes/degrees of freedom of the substructure can be eliminated thus reducing the size of the global matrices of the structure.

When a set of equations for structural problems, can be manipulated to collect the forces and displacements of one end of the substructure on one side of the equation and relate them to those on the other end with a matrix relation, that matrix is called the transfer matrix of the structure. The transfer matrix of a substructure is then multiplied by that of the neighboring structure, in contrast with the superposition that is used in conventional numerical methods. Thus, the matrix system that describes the dynamics of the structure becomes significantly smaller in size. The transfer matrix method becomes of even more appealing features, when identical substructures can be selected, thus, calculating the transfer matrix for one substructure is enough to describe all the dynamics of the whole structure. This particular feature is one that is inherent in all periodic structures by definition.

The investigation of the periodic structures was approached by different methods; the vast majority of literature applied the transfer matrix approach. The obtained transfer matrix is characterized by being simplistic when derived from a symmetric, conservative or non-conservative, dynamic stiffness matrix. The basic property of a simplistic matrix is that its eigenvalues appear in pairs one of which is the reciprocal of the other. This property of the transfer matrix has been looked at as one that introduces simplicity for the analysis; unfortunately, that same property causes the numerical instabilities in the analysis of structures with large number of cells.

Gadala et al. (1983) presented an early attempt for the formulation of a transfer matrix problem for a two-dimensional structure. The proposed model was used for a structure that could be divided into substructures in the form of strips whose nodes can be organized into two sets each lie on one side of the substructure. Due to complexity of the coupling between adjacent cells in two dimensional structures, the transfer matrix approach is not fully applicable. The analysis of the two-dimensional periodic structures has been primarily investigated through the graphical means of the propagation surfaces which were introduced by Mead and Parthan (1979).

In this study, the relation between the results obtained from the transfer matrix approach and those presented by the propagation surfaces will be studied in an effort to obtain better understanding of the propagation surfaces. Also, an attempt to produce propagation and attenuation curves to describe the dynamic characteristics of periodic plates will be introduced.

Static and dynamic deformations of machine tool holder play an important role in a machining process, which affecting the quality and productivity. Excessive chatter (self excited vibration) may cause tolerance violations. Cutting force and chatter models can be used to predict and overcome these problems. This would require dynamic data for the structures involved in a machining system (Montgomery and Altintas, 1991). These data are usually obtained by using mass -stiffness measurements and model analysis.

In this study, generalized equations are presented which can be used for predicting the static and dynamic properties of milling system components. Due to its wide use in industry, milling process is considered, however the same methods can be applied to other machining operations as well.

Modeling of milling process has been the subject of many studies some of which are summarized by Smith and Tlusty (1991). The focus of these studies has mostly been on the modeling of cutting geometry (Bayoumi et al., 1994; Min Wan et al., 2010). The mechanistic approach has been widely used for the force predictions and also has been extended to predict associated vibration of periodic system, (Faulkner and Hong, 1985; Langley 1996). It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam. The Euler Bernoulli model includes the strain energy due to the bending and the kinetic energy due to the lateral displacement.

In the present study, periodic elements are considered because these elements exhibit unique dynamic characteristics that make them act as mechanical filters for wave propagation (Mead, 1996; Doyle, 1997). As a result, waves can propagate along the periodic elements only within specific frequency bands called the ‘pass bands’ and wave propagation is completely blocked within other frequency bands called the ‘stop bands’. The ability of periodic structures to transmit waves from one location to another within the pass bands, can be greatly reduced when the ideal periodicity is disrupted resulting in the well-known phenomenon of localization.

Consider the vibration of an elastic tool holder, of length l and having unvarying circular cross sectional area in the xy-direction normal to the z-axis for vertical milling operation (figure 2). The vibration of the holder can be modeled as a shaft with one end at the bottom. The effects of the spindle motor are accounted for by including their collective inertia.

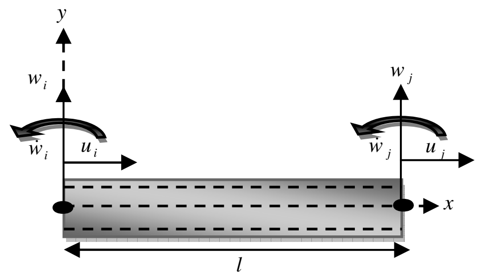


Fig. 2. Straight material cutting tool holder

The vibration of the tool holder performs not only a translator motion but also rotational motion. The angle of rotation, which is equal to the slope of the deflection curve, is expressed by w and the angular velocity and acceleration are $\dot{w}$ and $\ddot{w}$ respectively. The continuous model, shown in figure 2, has a flexible support representing tool/holder/spindle interface at one end. The governing equation motion for the beam based on the Euler–Bernoulli beam theory for each values of *x* in the interval from 0 to 1 will be:

$$EI\frac{\partial^4 w(x)}{\partial x^4}dx + EA\frac{\partial^2 u(x)}{\partial x^2} = 0 \tag{1}$$

Longitudinal translational displacements of the cutting tool holder normal to its length at each ends are denoted by w_i and w_j. While w' is the rotational displacement of endpoints, $w' = \partial w(x)/\partial x$. u_i and u_j are longitudinal displacements at each ends.

3. EQUATION OF MOTION AND BOUNDARY CONDITIONS

Since there are six nodal variables for the holder element, four for bending and two for the axial forces, a cubic polynomial function is assumed for *w(x),* and first order for *u(x)*. To consider the element which has three components at each end, w_i , w_i' and u_i at the top of the holder, w_j , w_j' and u_j at the bottom parallel to the surface of the machine table. For constant values of *EI* and *EA* equation (1) may be integrated to yield equations (2), Where c_i are constants of integration respect to x.

$$\begin{bmatrix} w_i \\ \dot{w}_i \\ w_j \\ \dot{w}_j \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & l^2 & l^3 \\ 0 & 1 & 2l & 3l^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = [A_w]\{c\} \qquad \begin{bmatrix} u_i \\ u_j \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix} \begin{bmatrix} c_5 \\ c_6 \end{bmatrix} = [A_u]\{c\} \tag{2}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \frac{1}{l^3}\begin{bmatrix} l^3 & 0 & 0 & 0 \\ 0 & l^3 & 0 & 0 \\ -3l & -2l^2 & 3l & -1l^2 \\ 2 & l & -2 & l \end{bmatrix}\begin{bmatrix} w_i \\ \dot{w}_i \\ w_j \\ \dot{w}_j \end{bmatrix} = [A_w^{-1}]\{w\}, \quad \begin{bmatrix} c_5 \\ c_6 \end{bmatrix} = \frac{1}{l}\begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_i \\ u_j \end{bmatrix} = [A_u^{-1}]\{u_{ij}\} \quad (3)$$

Using equation (3) to find the shape functions $\{N\}$, Where $\{N\} = [A^{-1}]\{w\}$*{N}*. Substitution of $\{N\}$values into the expressions of $w(x)$ and $u(x)$ yields the approximation of the mode shapes in the following equations.

$$w(x) = \left(1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}\right)w_i + \left(x - \frac{2x^2}{l} + \frac{x^3}{l^2}\right)w_i' + \left(\frac{3x^2}{l^2} - \frac{2x^3}{l^3}\right)w_j + \left(\frac{x^3}{l^2} - \frac{x^2}{l}\right)w_j' \quad (4)$$

$$u(x) = \left(1 - \frac{x}{l}\right)u_i + \left(\frac{x}{l}\right)u_j \quad (5)$$

3.1 Potential and Kinetic Energies

Consider the energy associated with approximation given by the previous equations (4) and (5). The potential energy *(PE)* of the tool holder is non-dimensionalized by *EI/l¸* will be expressible as:

$$PE_T = PE_w + PE_u = \frac{1}{2}\left[\left(\int_0^l EI[w''(x)]^2 dx\right) + \left(\int_0^l EA[u'(x)]^2 dx\right)\right] \quad (6)$$

Hence the vector $\{N'\} = \frac{\partial}{\partial x}\{N\}$, with entries $\{N_1'\}$ through $\{N_6'\}$, the first derivatives for equations (4) and (5) will be:

$$w'(x) = \left(\frac{6x^2}{l^3} - \frac{6x}{l^2}\right)w_i + \left(1 - \frac{4x}{l} + \frac{3x^2}{l^2}\right)w_i' + \left(\frac{6x}{l^2} - \frac{6x^2}{l^3}\right)w_j + \left(\frac{x^3}{l^2} - \frac{x^2}{l}\right)w_j' \quad (7)$$

$$u'(x) = \left(\frac{-1}{l}\right)u_i + \left(\frac{1}{l}\right)u_j \quad (8)$$

$$w''(x) = \left(\frac{12x}{l^3} - \frac{6}{l^2}\right)w_i + \left(\frac{6x}{l^2} - \frac{4}{l}\right)w_i' + \left(\frac{6x}{l^2} - \frac{12x}{l^3}\right)w_j + \left(\frac{6x}{l^2} - \frac{2}{l}\right)w_j' \quad (9)$$

Substitution of $\{N_w''\}$ and $\{N_u'\}$values into the expression of $w''(x)$ and $u'(x)$ yields the approximation of equation (10):

$$PE_T^e = \frac{EI}{2}\int_0^l \left[\left(\frac{12x}{l^3} - \frac{6}{l^2}\right)w_i + \left(\frac{6x}{l^2} - \frac{4}{l}\right)w_i' + \left(\frac{6x}{l^2} - \frac{12x}{l^3}\right)w_j + \left(\frac{6x}{l^2} - \frac{2}{l}\right)w'\right]^2 dx + \frac{EA}{2}\int_0^l \left(\frac{-1}{l}u_i + \frac{1}{l}u_j\right)^2 dx \quad (10)$$

The last expression can be recognized as proportional to the product of the transpose of the vectors *w* and *u*. assuming the holder rigidity *EI* and *EA* are constant within the elements. For each element the *wu*-stiffness matrix *K* is:

$$K_w{}^e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad \& \quad K_u{}^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{11}$$

That is

$$PE_T = \frac{1}{2}\left[\left(w^T K w\right) + \left(u^T K u\right)\right] \tag{12}$$

The kinetic energy of the element (*KE*) can be written in the alternative following form:

$$KE = KE_w + KE_u = \frac{m}{2}\left[\int_0^l [\dot{w}(x)]^2\,dx + \int_0^l [\dot{u}(x)]^2\,dx\right] \tag{13}$$

Considering the maximum kinetic energy at the end part of the holder $KE=KE_{max}$.
$\dot{w}(x)_w = \omega w(x)$ and $\dot{w}(x)_u = \omega u(x)$.

$$KE = \frac{m\omega^2}{2}\left[\int_0^l \left[\left(1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3}\right)w_i + \left(x-\frac{2x^2}{l}+\frac{x^3}{l^2}\right)w_i' + \left(\frac{3x^2}{l^2}-\frac{2x^3}{l^3}\right)w_j + \left(\frac{x^3}{l^2}-\frac{x^2}{l}\right)w_j'\right]^2 dx + \int_0^l \left[\left(1-\frac{x}{l}\right)u_i + \left(\frac{x}{l}\right)u_j\right]^2 dx\right] \tag{14}$$

For linear systems that obey Rayleigh's reciprocity principle, Srikantha Phani and Adhikari (2008), related the matrices *M* and *K* as follows:

$$K - \omega^2 M = 0 \tag{15}$$

Where ω is the natural frequency of an element. An eignvalue analysis has to be performed in designing a structural system that is to be subjected to dynamics forces. By substituting an eigenvalue λ_i into equation (15):

$$[K - \lambda_i M]w_i = 0 \quad \& \quad [K - \lambda_i M]u_i = 0 \tag{16}$$

Where eigenvectors w_i and u_i correspond to deflection mode that gives the shape of the element. Therefore, analysis of eignvalue equations gives important information on possible deflection modes experienced by the structure when it undergoes forces. In equation (16), since the mass matrix *(M)* is symmetric positive definite and stiffness matrix *(K)* are symmetric and either positive or positive semi-definite, the eigenvalues are all real and either positive or zero. The corresponding eigenvalue equations are having multiple eigenvalues. For an eigenvalue of multiplicity *N*, there are *N* vectors satisfying equation (16). The kinetic energy relative to the displacement will be:

$$KE_T = \frac{1}{2}\left[\left(w'^T M_w w'\right) + \left(u'^T M_u u'\right)\right] \tag{17}$$

Where *M* is the mass matrix for the system elements and defined by:

$$M_w^{\ e} = \frac{m}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad \& \quad M_u^{\ e} = \frac{m}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{18}$$

Using equations (12) and (18) the dynamic equations becomes:

$$\frac{m}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}\begin{bmatrix} w_i'' \\ w_i''' \\ w_j'' \\ w_j'' \end{bmatrix} + \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}\begin{bmatrix} w_i \\ w_i \\ w_j \\ w_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

$$\frac{m}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} u_i'' \\ u_j'' \end{bmatrix} + \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_i \\ u_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{20}$$

3.2 Shape Function of Periodic Elements

One element for the cutting tool holder model gives inaccurate results if higher modes are excited, therefore, more elements must be used to model the entire structure. If multiple elements are used, equations for all elements must be assembled into a model of entire structure as a whole. For dynamic analysis of the holder and merging equations (5) and (6), the deflection is interpolated within a holder element as:

$$wu_{ij} = \left(1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}\right)w_i + \left(x - \frac{2x^2}{l} + \frac{x^3}{l^2}\right)w_i' + \left(1 - \frac{x}{l}\right)u_i + \left(\frac{3x^2}{l^2} - \frac{2x^3}{l^3}\right)w_j + \left(\frac{x^3}{l^2} - \frac{x^2}{l}\right)w_j' + \left(\frac{x}{l}\right)u_j \tag{21}$$

4. MODEL OF VIBRATING TOOL HOLDER

It is convenient to have formulations of motion which makes use of quantities relating to the whole system from which elements are made up. The equations of motion can be obtained from the preceding expressions for kinetic *KE* and potential *PE* energies using the variation or Lagrangian approach. Combining equations (11) and (14) for getting the total energies. The equations of motion for the vibratory system can be given in the structure as:

$$\sum_{e=1}^{n}[M_e]\{\ddot{\delta}_e\} + \sum_{e=1}^{n}[K_e]\{\delta_e\} = \sum_{e=1}^{n}\{F_e\} \tag{22}$$

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{F\} \tag{23}$$

The value of vibration expressed as:

$$\{\delta\} = \left[\{F\} - [M]\{\ddot{\delta}\}/[K]\right] \tag{24}$$

Where $\{\delta_e\}=\{w_1 \quad w_1' \quad u_1 \quad .... \quad w_N \quad w_N' \quad u_N\}$ is nodal deflection vector of the element, *n* denoting number of nodal points and $\{F_e\}$ is the vector of external forces. Taking the cutting tool holder path as circular arc moves by the feed per tooth (chip load), c, in case of up milling tooth 1 in position ϕ_1 engages over the arc of cut, where $\phi_S \langle \theta_1 \langle \theta_E$ and $\phi_2=\phi_1+\pi/2$. The force acting on the holder can be added and reflected into the F_u and F_w components in the tool axis:

$$F_u = K_S A_d c[\sin(\phi)\cos(\phi)+0.3\sin^2(\phi)] \tag{25}$$

$$F_w = K_S A_d c[\sin^2(\phi)-0.3\sin(\phi)\cos(\phi)] \tag{26}$$

$$F=\sum_{n=1}^{N} F_u + \sum_{n=1}^{N} F_w \tag{27}$$

Both force components are periodic in 2ϕ, where $\phi=2\pi(N/60)t$.

4.1 Longitudinal Vibration

Consider three elements model of the longitudinal vibration and with one degree of freedom as shown in figure 3. Since the three cells of the system with two different materials combinations (spring steel-rubber and spring steel copper) are rigid and rotating at the same time with one angle. Each element of the model has a kinetic and potential energy. The integral my take various forms for the tool holder. The flexural rigidity *EI* of the element must be taken into account. From equation (20) three sets of matrices and their corresponding with identical equations and different sets of unknown modal displacements u_i, can be assembled together by superimposing them to yield equation (28) in the form:

$$\frac{m}{18}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} u_1'' \\ u_2'' \\ u_3'' \\ u_4'' \end{bmatrix} + \frac{3EA}{l}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

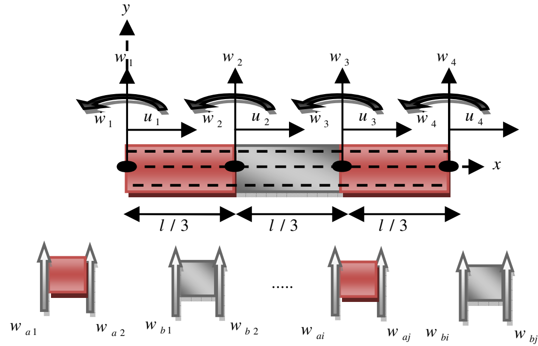


Fig. 3. Periodic cutting tool holder model

4.2 Translation and Rotational Vibration

The global mass and stiffness matrices for a clamped holder at the spindle and free with the cutting tool is shown in figure 1. Using three elements and four nodes with *l=l/3*, the equations for the finite element at *(i=1, j=2)*, *(i=2, j=3)*, *(i=3, j=4)*, becomes:

$$\frac{m}{1260}\begin{bmatrix} 156 & \frac{22l}{3} & 54 & \frac{-13l}{3} \\ \frac{22l}{3} & \frac{4l^2}{9} & \frac{13l}{3} & \frac{-l^2}{3} \\ 54 & \frac{13l}{3} & 156 & \frac{-22l}{3} \\ \frac{-13l}{3} & \frac{-l^2}{3} & \frac{-22l}{3} & \frac{4l^2}{9} \end{bmatrix}\begin{bmatrix} \ddot{w}_i \\ \ddot{w}_i \\ \ddot{w}_j \\ \ddot{w}_j \end{bmatrix}+\frac{9EI}{l^3}\begin{bmatrix} 12 & 2l & -12 & 2l \\ 2l & \frac{4l^2}{9} & -2l & \frac{2l^2}{9} \\ -12 & -2l & 12 & -2l \\ 2l & \frac{2l^2}{9} & -2l & \frac{4l^2}{9} \end{bmatrix}\begin{bmatrix} w_i \\ w_i \\ w_j \\ w_j \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{29}$$

General structure of equations of motion is: $M\begin{bmatrix} w_i'' \\ w_i''' \\ u_i'' \\ w_j'' \\ w_j''' \\ u_j'' \end{bmatrix}+K\begin{bmatrix} w_i \\ w_i \\ u_i \\ w_j \\ w_j \\ u_j \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$

General structure of equations of motion for three elements periodic holder:

$$M=\begin{bmatrix} M_c+M_a & M_b & 0 \\ M_b^T & M_c+M_a & M_b \\ 0 & M_b^T & M_c \end{bmatrix}, \qquad K=\begin{bmatrix} K_c+K_a & K_b & 0 \\ K_b^T & K_c+K_a & K_b \\ 0 & K_b^T & K_c \end{bmatrix} \tag{30}$$

$$M_a=\begin{bmatrix} 156 & \frac{22l}{3} \\ \frac{22l}{3} & \frac{8l^2}{9} \end{bmatrix}, M_b=\begin{bmatrix} 54 & \frac{-13l}{3} \\ \frac{13l}{3} & \frac{-l^2}{9} \end{bmatrix}, M_c=\begin{bmatrix} 156 & \frac{-22l}{3} \\ \frac{-22l}{3} & \frac{8l^2}{9} \end{bmatrix}, K_a=\begin{bmatrix} 12 & 2l \\ 2l & \frac{4l^2}{9} \end{bmatrix}, K_b=\begin{bmatrix} -12 & 2l \\ -2l & \frac{2l^2}{9} \end{bmatrix},$$

$$K_c=\begin{bmatrix} 12 & -2l \\ -2l & \frac{4l^2}{9} \end{bmatrix}$$

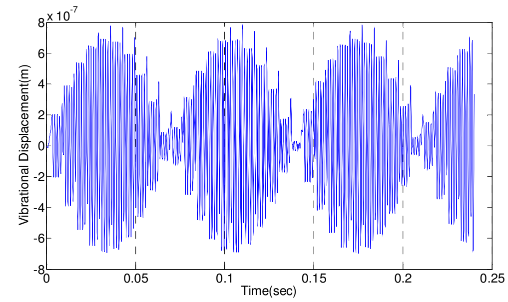
5. SIMULATION RESULTS

It is important to know that a harmonic force produces harmonic vibrations of the same frequency, and the amplitude of the vibrations depends on the amplitude of the cutting force and on the ratio of the frequency of the force over the natural frequency of the system. If the two frequencies are equal, the case of resonance and maximum vibration amplitude will be occurring.

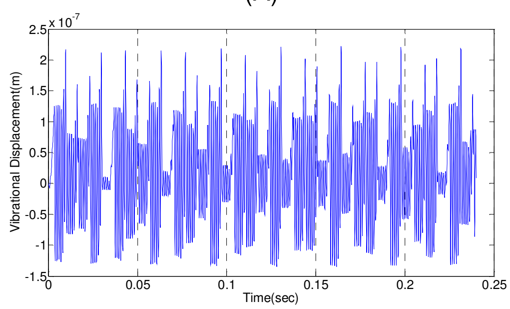
In this paper all simulations and illustrations are based on end mill with helical smooth edges using the proposed dynamic milling model with parameters listed in table 1, based on numerical theory and technique with Eulerian approach (Jalili Saffar et al., 2008). The analysis of the cutting force vibration and its effects on forced variation is plotted in the set diagrams in figure 4, which contain time plots that illustrate the stability improvements from straight to periodic tool holders, with four homogeneous teeth and various cutting engagement. The plots were obtained from the computer program written in MATLAB.

Table 1 . Milling Simulation Parameters

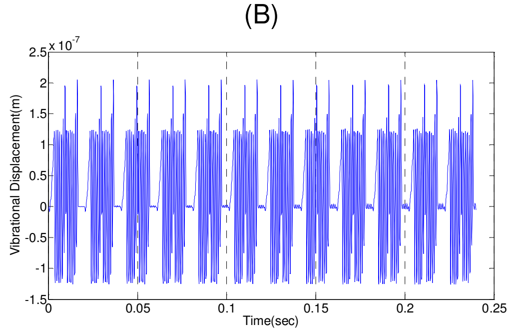
Specific Cutting Force, *Ks*	2100 N/mm
Nominal feed per tooth, f_T	2 mm
Cutting tool diameter, *D*	16 mm
Too holder diameter, D_h	24 mm
Number of cutting edges, *Z*	4
Axial depth of cut, A_d	5 mm
Radial depth of cut, R_d	3 mm
Spindle speed, *N*	3000 rpm
Number of samples, *NS*	1000
Modulus of Elasticity for Rubber, *E*	0.1 GPa
Modulus of Elasticity for Copper, *E*	117 GPa
Modulus of Elasticity for spring steel, *E*	210 GPa



(A)



(B)



(C)

Fig. 4. Model simulation results of vibration patterns, (A) periodic spring steel-rubber, (B) periodic spring steel-copper and (C) straight spring Steel of milling tool holder.

The program follow the rotation of the cutter in 250 steps/revolution, *dfi=360/240=1.44°*, and it runs for 1000 steps, that is 4 revolutions. The tooth passing frequency is *NZ*, where *N* denoted the rotational speed and *Z* is the number of cutting edges of the milling cutter. The time between two consecutive cuts (*T*) causes a phase difference as *T=1/NZ*. The feed per tooth (f_T) coupled with a variable spindle speed *(N)* in a changing feed rate (*f*) which causes modulation of the cutting forces F_u and F_w. During several initial tooth periods, vibrations start to develop and then reach the steady state in which the vibration at total time (*t*) is determined, where *t = 0.25* sec, as shown in the following figure 4.

The resultant cutting force of all simulations under the same cutting parameters of table 1, are in good agreement, as shown in figure 5. Waiting for vibration to settle look for the cutting force in the last revolution, i.e. the last 250 steps, just for the purpose of plotting this part, as shown in figure 6.

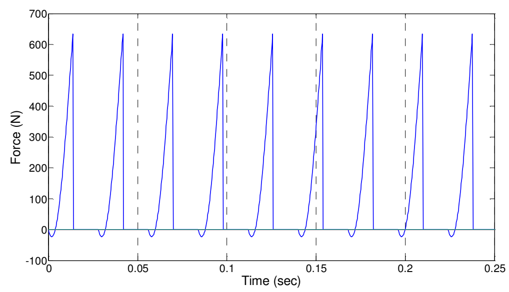


Fig. 5. Milling component force on cutter with 4 straight teeth and 0.25 sec cutting time

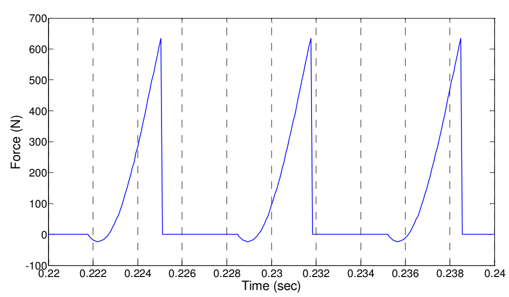


Fig. 6. Force in steady state for last revolution

6. CONCLUSION

Vibration is generally avoided either by stiffening the relative compliance between the cutting tool system and workpiece, or by reducing the axial and radial depths of cut. In this paper, a new approach for monitoring vibration during the machining process by regulating the periodic materials of the machine tool holder is presented. A digital dynamic simulation model was proposed to investigate the influence of periodic cutting tool holders as well as structural parameters on the stability of milling vibrations. The model written in MATLAB includes the contribution of the mass and stiffness and its affect on the cutting force amplitudes. The paper presents a new class of periodic machine tool holder system for isolating the vibration transmission from cutting tool holder to the machine tool table in an attempt to produce a quiet surface finish. A theoretical model is developed to describe the dynamics of wave propagation in a periodic tool holder. The model is derived using the theory of finite elements. The model of three periodic elements, spring steel-rubber, spring steel-copper and straight spring steel to compute the vibration amplitudes and forces are presented. The transfer matrix formulation for each element is given.

A comparison between those theoretical approaches with real measurements will be studied in the next investigation.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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