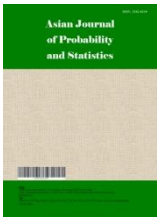

Asian Journal of Probability and Statistics

15(1): 1-7, 2021; Article no.AJPAS.75096
ISSN: 2582-0230

On Sequential Probability Sampling Plan for a Truncated Life Tests Using Rayleigh Distribution

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2021/v15i130339
Editor(s):
(1) Dr. Manuel Alberto M. Ferreira, Lisbon University, Portugal.
Reviewers:
(1) Lishamol Tomy, Deva Matha College Kuravilangad, India.
(2) Chun, Pamson Bentse, Plateau State University, Nigeria.
Complete Peer review History: https://www.sdiarticle4.com/review-history/75096

Original Research Article

Received 19 July 2021
Accepted 29 September 2021
Published 01 October 2021

Abstract

This paper proposed a sequential probability sampling plan for a truncated life test using a Rayleigh distribution from a designed double sampling plans where the interest was to obtain the minimum sample size necessary to assure that the average life time of a product is longer than the default life time at the specified consumer's and producer's confidence level. Estimations of minimum sample, acceptance and rejection numbers obtained are analyzed and presented to explain the usefulness of sequential plans in relation to single and double sampling plan. Probability of acceptance (Pa), Average sample number (ASN) and Average outgoing quality (AOQ) for the plans are computed. The three regions; acceptance, continue sampling and rejection were determined. The five points necessary to plot ASN curve were also computed and presented.

Keywords: Average Sample Number (ASN); Acceptable Quality Level (AQL); consumer's risk; producer's risk; Lot Tolerance Percent Defective (LTPD).

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1 Introduction

Sequential probability acceptance sampling is an important quality control tool in terms of making decisions about a particular lot. As companies all over the world are taking improvement quality of their product to be uttermost, it is not surprise that study on sequential acceptance sampling is paramount. Acceptance sampling plans are important tools widely used for promoting product quality in the industries. It is an inspection procedure concerned with accepting or rejecting a given lot of large size of products on the basis of its quality after inspection of a sample taken from the lot [1].

The single-sampling plan is basic to all acceptance sampling. The simplest form of such a plan is single sampling by attributes, which relates to dichotomous situations, that is, those in which the inspection results can be classified into only two classes of outcomes. This includes go/no-go gauging procedures as well as other classifications, such as measurements in or out specifications. Applicable to all sampling situations, the attributes single-sampling plan has become the bench mark against which other sampling plans are judged [2]. Double- and multiple-sampling plans reflect the tendency of many experienced inspectors to give a questionable lot an additional chance. Thus, in double sampling, if the results of the first sample are not definitive in leading with acceptance or rejection, a second sample is taken that then leads to a decision on the disposition of the lot. This approach makes sense, not only as a result of experience but also in the mathematical properties of the procedure. For one thing, the ASN can usually be made to be less for a double-sampling plan than for a single-sampling plan with the same protection [3]. Sequential sampling is an extension of the double-sampling and multiple-sampling concept. In sequential sampling, we take a sequence of samples from the lot and allow the number of samples to be determined entirely by the results of the sampling process. In practice, sequential sampling can theoretically continue indefinitely, until the lot is inspected 100%. In practice, sequential sampling plans are usually truncated after the number inspected is equal to three times the number that would have been inspected using a corresponding single sampling plan. This approach allows one to draw conclusions during the data collection, and a final conclusion can possibly be reached at a much earlier stage as is the case in Classical Hypothesis Testing.

Since the 18th century there has been a growing interest in statistical hypothesis testing. In literatures, several theories have been already proposed to extend the applicability of the mathematical background or to optimize calculation [4].

Double acceptance sampling was developed for a truncated life test based on the inverse Rayleigh distribution [5] the aim was to obtain a minimum sample necessary to ensure the specified average life time under a given consumer's risk. The probability of acceptance using the cumulative density function of inverse Rayleigh distribution is calculated for different consumer's confidence levels with fixed producer's risk. In a similar research, double sampling plan based on truncated life test was developed [6] assuming the lifetime follows the Rayleigh distribution. Probability of acceptance is calculated for different consumer's confidence levels fixing the producer's risk at 0.05. Probability of acceptance and producer's risk are discussed with the help of Tables and examples. Also a similar study by [7] developed group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and Log-Logistics distributions. In all these works the author's concern was to obtain the minimum sample to be inspected so that both the consumer and producer will be happy in the end. The sequential sampling always gives the minimum sample size hence the need for this research.

Various sampling plans and sequential tests have also been proposed for testing different hypotheses since [8]. For example, [5] in their research on Double Acceptance Sampling Plans Based on Truncated Life Tests for the inverse Rayleigh distribution (IRD), Double acceptance sampling plans was developed using the IRD as a model for the life time random variable for a truncated life test. In their paper, the single acceptance sampling developed by [9] was extended to double acceptance sampling in terms of finding the minimum number of sample sizes necessary to ensure the specified average life time under a given consumer's risk. The probability of acceptance using the cumulative distribution function of IRD was calculated for different consumer's confidence levels with fixed producer's risk. The operating characteristic curves of the sampling plans as well as the tables were presented.

A universal sequential test for detecting outliers among all collected observation sequences was constructed by [10], also [11] in their paper uses a well-known test procedure of statistical science called as Sequential

Probability Ratio Test (SPRT) is adopted for Burr Type XII model in assessing the reliability of developed software. In their research, SPRT requires considerably less number of observations when compared with the other existing testing procedures. Hence Sequential Analysis of Statistical Science could be adopted to decide upon the reliability or otherwise of the developed software very quickly. Generally, The ability to potentially reduce the sample size required to make a decision in an experiment has numerous applications because it leads to the conserving of resources, making funding easier to appropriate [12].

The main objective of this paper was to determine the minimum number of sample size required to run an inspection and make a good decision for lots using Rayleigh distribution under sequential probability sampling plan and compare the proposed plan to that of double sampling plan.

2 Methodology

The concept of sequential decision making after each item being inspected is based on the SPRT developed by [8]. The acceptance sampling is basically a test of hypothesis:

H_0: $p = p_1$ for good quality

Against an alternative

H_1: $p = p_2$ for poor quality

A sequential probability sampling (SPS) is a hypothesis test for sequential samples. Sequential sampling works in a very non-traditional way; instead of a fixed sample size, you choose one item (or a few) at a time, and then test your hypothesis. You can either:

- Reject the null hypothesis (H_0) in favor of the alternate hypothesis (H_1) and stop,
- Keep the null hypothesis and stop,
- Fail to reach either conclusion and continue sampling.

If you fail to reach a conclusion, you repeat the sampling and then the hypothesis test. You keep on repeating this process until you have a sound conclusion, so you don't know the how big your sample will be until you're finished testing [13].

If ϕ is the event of having found *d* defectives after *n* items have been examined, the conditional probabilities of ϕ are:

$$\begin{aligned} \Pr(\phi \mid H_0) &= p_1^d (1-p_1)^{n-d} = x \\ \Pr(\phi \mid H_1) &= p_2^d (1-p_2)^{n-d} = y \end{aligned} \tag{1}$$

If the quality of the product offered is p_1, the lot should be accepted. For an acceptable lot

$$\begin{aligned} x &\geq 1-\alpha \\ y &\leq \beta \end{aligned} \tag{2}$$

It should be noted that here, the p is a function of the Rayleigh distribution. As it is assumed that the life time of a product follows the Rayleigh distribution with the Cumulative Density Function (CDF) given by

$$F(t,\lambda) = 1-e^{-\frac{t}{\lambda}} \quad t>0, \lambda>0 \tag{3}$$

To determine the region for decision, we consider the following areas

$$\begin{aligned} &i) -h_1 + sn < x < h_2 + sn \\ &ii) x_a \leq -h_1 + sn \\ &iii) x_r \geq h_2 + sn \end{aligned} \tag{4}$$

$$h_1 = \log(1-\alpha)/\beta \big/ \left(\log(p_2/p_1) + \log(1-p_1)/(1-p_2)\right)$$
$$h_2 = \log(1-\beta)/\alpha \big/ \left(\log(p_2/p_1) + \log(1-p_1)/(1-p_2)\right) \quad (5)$$
$$s = \log(1-p_1)/(1-p_2) \big/ \left(\log(p_2/p_1) + \log(1-p_1)/(1-p_2)\right)$$

3 Results

The three regions that are represented in the following graphs; Fig. 1 through Fig. 3 are computed from Eqn. 3 we obtain the following lines:

$$x_a = -h_1 + sn = -2.9226 + 0.1103n$$
$$x_r = h_2 + sn = 1.9162 + 0.1103n \quad (6)$$

Note that we are comparing the proposed plan with that of single sampling plan (SSP) and double sampling plan (DSP) as proposed by [1] and [5].

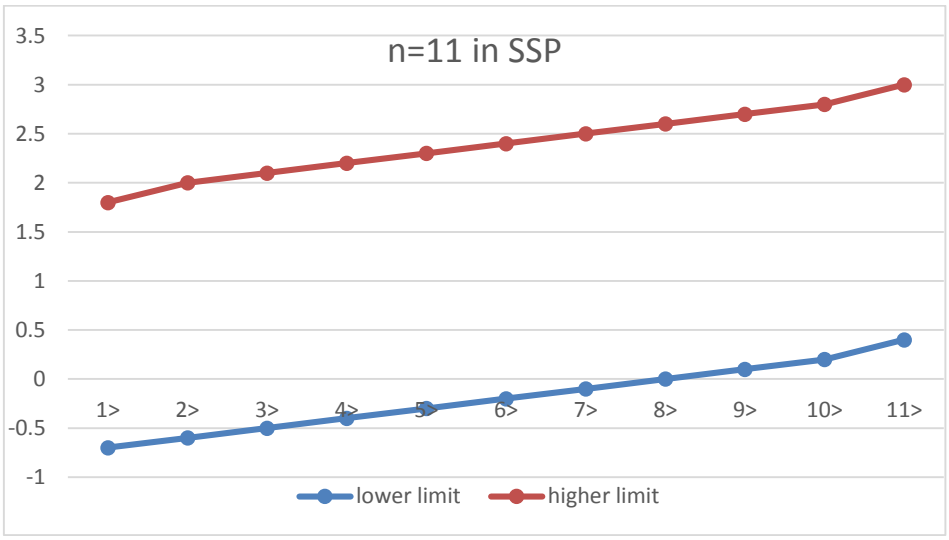

Fig. 1. Minimum sample to be inspected is 8 Compared with 11 in SSP

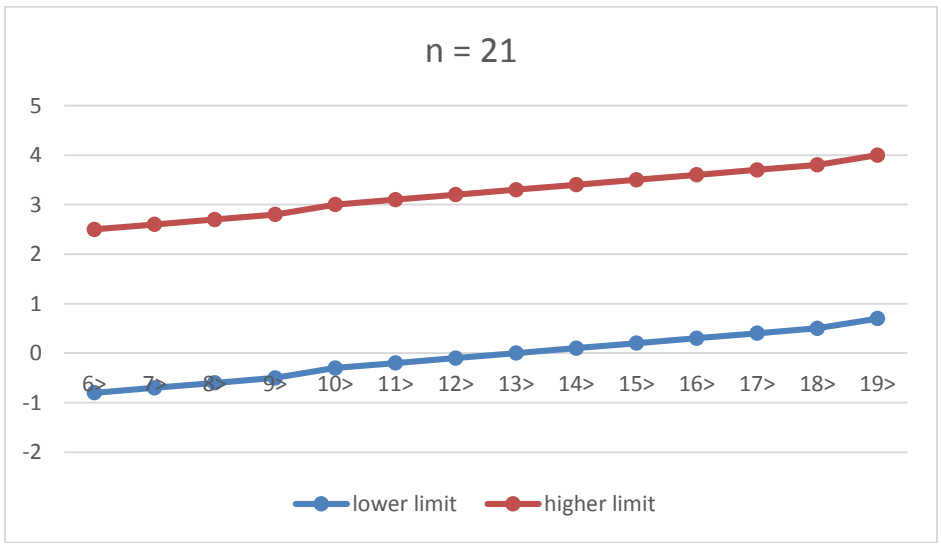

Fig. 2. Minimum sample to warrant a decision is 13 compared with 21 in SSP

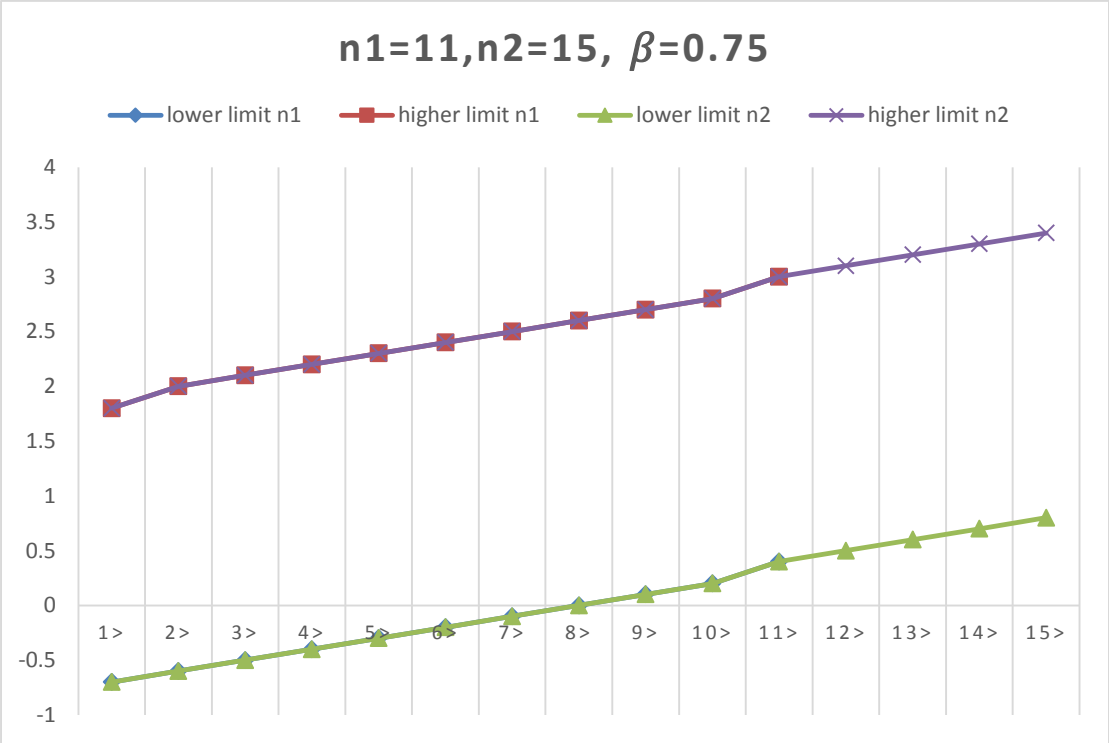

Fig. 3. Minimum sample to be inspected is 8 Compared with 15 in DSP

Table 1. Sequential probability ratios for single sampling plan

Trial	Lower limit		-*-	Higher limit	
1>	-0.7	continue	-*-	1.8	continue
2>	-0.6	continue	-*-	2	97.9%
3>	-0.5	continue	-*-	2.1	69%
4>	-0.4	continue	-*-	2.2	54.5%
5>	-0.3	continue	-*-	2.3	45.8%
6>	-0.2	continue	-*-	2.4	40%
7>	-0.1	continue	-*-	2.5	35.9%
8>	0	0.3%	-*-	2.6	32.8%
9>	0.1	1.5%	-*-	2.7	30.3%

Table 2. Sequential probability ratios for double sampling plan

Trial	Lower limit		-*-	Higher limit	
6>	-0.8	continue	-*-	2.5	41.9%
7>	-0.7	continue	-*-	2.6	37.5%
8>	-0.6	continue	-*-	2.7	34.2%
9>	-0.5	continue	-*-	2.8	31.6%
10>	-0.3	continue	-*-	3	29.6%
11>	-0.2	continue	-*-	3.1	27.9%
12>	-0.1	continue	-*-	3.2	26.5%
13>	-0	continue	-*-	3.3	25.3%
14>	0.1	0.7%	-*-	3.4	24.3%
15>	0.2	1.4%	-*-	3.5	23.4%

The result obtained from Table 3 is represented in Table 4. It is clear that the proposed plan yielded the minimum sample to be inspected than the existing double sampling plan by [6].

Table 3. The Average Sample Number (ASN)

P	ASN
0	h_1/s
P_1	$(1-\alpha)h_1 - \alpha h_2/(s-p_1)$
S	$h_1h_2/s(1-s)$
P_2	$(1-\beta)h_2 - \beta h_1/p_1 - s$
1	$h_2/1-s$

Table 4. The ASN and Probability of acceptance

P	ASN	Prob. Acceptance
0	8	100
P_1	1	0.9309
s	17	0.6698
P_2	15	0.20
1	2	0

4 Discussion

The sequential sampling plan is described as shown in Figs. 1 – 3 and the minimum number of items to be inspected is tabulated and shown in Tables 1 and 2 whereas the probability of acceptance of its respective sample size is shown in Table 3. The two lines from Figs. 1-3 indicated the acceptance (lower limit) and rejection (higher limit) lines, these lines were computed from Eqn. 4 and plotted on the x and y axis. The region in-between these lines are called no decision region as such continues inspection is necessary. Table 1 shows that from item 1 to 7 it is recommended to continue sampling, one can decide to accept such lot when the 8th items is inspected and no any defective item is found. Comparing with the plan by DSP where the minimum number of inspection is given as 11. Table 2 gave a minimum value to warrant a decision to be 13 compared with that of a plan by DSP as reported by [1] they got 21 as the minimum value. The probability of acceptance as per the proposed plan using the Rayleigh distribution is presented in Table 3.

5 Conclusion

In conclusion we are able to show that the sequential probability method gives the minimum number of inspection units to warrant a good decision making. Both the single sampling plan and double sampling plan yielded a value greater than that of sequential sampling plan. This will protect both the consumer and producer from making a hasty decision that may in the long run jeopardize the good relationship enjoyed by them.

Competing Interests

Authors have declared that no competing interests exist.

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Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
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