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Modeling the Effect of Random Environmental Perturbation on Data Precision in Niger Delta Crude Oil Production

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

Decision-makers most times rely on data precision to support their decision-making processes. There is also a strong belief, however, that data quality or data precision problems are widespread in practice and that reliance on poor data quality or wrong data sets can lead to devastating consequences on crude oil production and financing. This study applies the Runge-Kutta numerical method of order 45 (ordinary differential

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equation of order 45) to examine the effect of data precision index on the Niger Delta crude oil production due to the variation of the initial condition. The study has found out that the initial condition four (4), (IC 4) with a data precision index value of 0.0118 ranked best in the first crude oil well, the initial condition (IC) 15 with a data precision index value of 0.0021 ranked best in the second crude oil well, the initial condition (IC) 17 with a data precision index value of 0.0021 ranked best in the third crude oil well and the initial condition (IC) 20 with a data precision index value of 0.0234 ranked best in the fourth crude oil well. The study has observed that each crude oil data is associated with a best-fit-value of the data precision index.

Keywords: Data precision; numerical method; Niger Delta; crude oil; random environmental; perturbation.

1 Introduction

The investigation of the consequences of oil spill on sea-food safety in coastal areas of Ibeno, Akwa-Ibom, State observed the main concentration of Total Petroleum Hydrocarbons (TPH) in the tissues of various fish species sampled to be increasing as a result of oil spills [1], Izevbizua and Apanapudor, 2020, Izevbizua and Apanapudor, [2] Apanapudor, et al, [3]. Similarly, the evaluation of the historical influence of petroleum activities on various episodes of economic crises in Nigeria, using a descriptive technique to analyse data obtained from secondary sources and affirming that transmogrification of the economy from agricultural-based to petroleum-based laid the foundation for the current economic crises in Nigeria according to Paul [4]. Also, while exploring the extent of the environmental degradation in the Niger Delta region and examining the efforts of oil companies in remediating the degraded farmlands in the Niger Delta, finds that oil pollution causes damage to human health, agricultural land and fish ponds as well as long-standing ecological malfunctioning [5-8].

Examining the effects of environmental degradation on human health in nine selected oil communities in Delta State, Nigeria using cluster and principal component analysis, employed both primary and secondary data; primary data was collected through administration of questionnaires while secondary data was from archival records of in-and-out patients from the government hospitals or clinics located in Okpai, Kwale, Benekuku, Ubeji, Bomadi, Ekakprame, Erhoike, Afiesere, Uzere and its environs for a year. One-year hospital data was used based on data availability and consistency on the required ailments such as bronchitis, cough, asthma, cardiovascular diseases, eye infection and skin infection in order to ascertain the effect of gas flaring on human health. One important observation in the study is that gas flaring has a statistically significant, but dangerous impact on human health in the affected areas giving the high temperature and emission to the atmosphere [9,10].

Petroleum pipelines vandalization and illegal bunkering is a major problem to aquatic oil spillage and degradation of the environment. Research observes that oftentimes illegal bunkering and petroleum pipeline vandalization results from destructive tendencies of restive youths, who are aggrieved by government neglect of oil producing communities and corruption of the ruling class in amassing wealth through collaborations with oil companies. Unfortunately, these social vices perpetrated by the youths have a counter-effect in increasing the levels of oil spill in the environment and the negative effect on aquatic and land agricultural produce by Boris [11]. Also, the analysis of the influence of petroleum on Nigerian economy using secondary annual data from 2000 to 2009 Ogbonna & Appah-Ebimobowei, [12] Aderibigbe, and Apanapudor, [13] Izevbizua and Apanapudor, [2] has a significant story to tell about the Nigerian economy. Linear regression models, as tools unveiled that petroleum has substantial direct influence on the economy. Unfortunately, the mismanagement of the proceeds from petroleum exploration imbues Nigeria into the resource curse dilemma [14,15]. This can be seen as the critical assess of the effect of oil exploration on poverty in the Niger Delta region of Nigeria. The authors' extensive review of the literature combined with drawing conclusions from the empirical findings, restates the neglect of the region and the consequences of pollution as a drawback to economic progress. The study further concludes that the greatest negative tendency associated with the exploration and exploitation of oil in this region is environmental degradation [16,17], Ezimadu, et al , 2020). The examination of the environmental impact of oil exploration and exploitation in Niger Delta of Nigeria, using tabular analysis of data obtained from secondary sources. The study finds that the oil industry sited within this region has contributed enormously to the economic growth of the country but unsustainable oil exploration activities have rendered the Niger Delta region one of the five most severely damaged ecosystems in the World [18]. Assessment of oil exploration and spillage in the Niger Delta region of Nigeria, using comparative analysis of secondary data covering periods from 1976 to 2000 on descriptive techniques such as line and bar graphs, and found a decrease in oil spillage quantity but an increase in the number and times of oil spill [19,10] and decrease in crude oil reserve Statista, [20,21]. However, there are yet untapped crude oil reserves in the deep sea, Zoetalents (2024).

The examination of the effects of oil pollution on crop production in Rivers State, Nigeria on a sample of 296 respondents drawn from 17 out of 23 Local Government Areas, applied a stochastic trans-log production function in a multi-stage sampling technique, the results indicate that the effect of crude oil pollution on crop farms reduced the size of farmland, significantly at 1%, reducing marginal physical product (MPP), while in non-polluted farms output increased. Physical inputs, crude oil pollution variables and their interactions show strong negative (diminishing) returns to scale in oil polluted farms, but in non-polluted farmlands results indicate strong positive returns to scale. The technical efficiency results show that less than 22% of crop farmers were over 80% efficient in their use of resources in oil polluted farmlands, while technical efficiency in non-polluted farmlands indicates a high efficiency of 33% [22,23,24]. Given the present circumstances, in the Niger Delta and the need for improved economic activities for the population, it becomes very imperative for studies to explore the impact of environmental degradation on specific issues such as aquatic production to enable policy makers pin-point areas of concentration in the implementation of various policies for the economic development of the region. Worgu [25] Okwonu, et al. [26] Aderibigbe and Apanapudor [13] Izevbizua and Apanapudor [27] noted that oil spill in rural communities such as Kalabari region can have serious effects on lives of residents in those communities. Oil spill affects land fertility, agriculture in different aspects. According to Islam and Tanka [28]. The disaster of oil spills can be effective based on the quantity of oil spill that floats to the river. Elum et al. [29] Okwonu, et al. [30] Okwonu and Apanapudor [10] observed that oil spill and the attendant youth restiveness resulting from deprivation, affects the socio economic environment and infrastructure.

2 Materials and Methods

In this paper, we shall focus on a method of analyzing the research questions that we have proposed using a mathematical formulation and mathematical preliminary.

2.1 Simplifying assumptions

Here, we state the following simplifying assumptions as a guide to formulating a set of mathematical equations describing the production of crude oil over time.

i. The rate of change of crude oil that is produced is directly proportional to the quantity of crude oil produced overtime.
ii. The rate of change of crude oil that is produced is inhibited by the intra- competition coefficient.
iii. The initial quantity of crude oil that is produced at the initial time is specified to be positive.
iv. The duration of crude oil that is produced is assumed to be on a monthly basis which is considered with every twenty-five (25) months of crude oil production timing.

2.2 Mathematical formulations

Motivated by the work for Dehghan and Heris [31] Offor (2020), Apanapudor, et al. [32] Okwonu, et al. [26] Izevbizua and Apanpudor [15] we consider a dynamical system consisting of four nonlinear ordinary differential equations having the following mathematical structure.

$$\begin{cases} \frac{dc_1}{dt} = a_1c_1 - b_1c_1^2 - r_1c_1c_{2-}r_2c_1c_3 - r_3c_1c_4 \\ \frac{dc_2}{dt} = a_2c_2 - b_2c_2^2 - r_4c_2c_{1-}r_5c_2c_3 - r_6c_2c_4 \\ \frac{dc_3}{dt} = a_3c_3 - b_3c_3^2 - r_7c_3c_1 - r_8c_3c_2 - r_9c_{21}c_4 \\ \frac{dc_4}{dx} = a_4c_4 - b_4c_4^2 - r_{10}c_4c_2 - r_{11}c_4c_2 - r_{12}c_4c_3 \end{cases} \qquad (1\text{-}4)$$

Manafian [33] with non-negative initial conditions

From (1), at a steady state,

$$\frac{dc_1}{dt} = 0$$

$$\Rightarrow a_1c_1 - b_1c_1^2 - r_1c_1c_2 - r_2c_1c_3 - r_3c_1c_4 = 0 \quad (5)$$

From (2), at a steady state,

$$\frac{dc_2}{dt} = 0$$

$$\Rightarrow a_2c_2 - b_2c_2^2 - r_4c_2c_1 - r_5c_2c_3 - r_6c_4 = 0 \quad (6)$$

From (3), at a steady state,

$$\frac{dc_3}{dt} = 0$$

$$\Rightarrow a_3c_3 - b_3c_3^2 - r_7c_3c_1 - r_8c_3c_2 - r_9c_3c_4 = 0 \quad (7)$$

From (4), at a steady state,

$$\frac{dc_4}{dt} = 0$$

$$\Rightarrow a_4c_4 - b_4c_4^2 - r_{10}c_4c_1 - r_{11}c_4c_2 - r_{12}c_4c_3 = 0 \quad (8)$$

From equation (5), we have

$c_1(a_1 - b_1c_1 - r_1c_2 - r_2c_3 - r_3c_4) = 0$
$\Rightarrow c_1 = 0$ or $a_1 - b_1c_1 - r_1c_2 - r_2c_3 - r_3c_4 = 0$
$\Rightarrow b_1c_1 + r_1c_2 + r_2c_3 + r_3c_4 = a_1$ (9)

From equation (6), we have

$c_2(a_2 - b_2c_2 - r_4c_1 - r_5c_3 - r_6c_4) = 0$
$\Rightarrow c_2 = 0$ or $a_2 - b_2c_2 - r_4c_1 - r_5c_3 - r_6c_4 = 0$
$\Rightarrow b_2c_2 + r_4c_1 + r_5c_3 + r_6c_4 = a_2$ (10)

From equation (7), we have

$c_3(a_3 - b_3c_3 - r_7c_1 - r_8c_2 - r_9c_4) = 0$
$\Rightarrow c_3 = 0$ or $a_3 - b_3c_3 - r_7c_1 - r_8c_2 - r_9c_4 = 0$
$\Rightarrow b_3c_3 + r_7c_1 + r_8c_2 + r_9c_4 = a_3$ (11)

From equation (8), we have

$c_4(a_4 - b_4c_4 - r_{10}c_1 - r_{11}c_2 - r_{12}c_3) = 0$
$\Rightarrow c_4 = 0$ or $a_4 - b_4c_4 - r_{10}c_1 - r_{11}c_2 - r_{12}c_3 = 0$
$\Rightarrow b_4c_4 + r_{10}c_1 + r_{11}c_2 + r_{12}c_3 = a_4$ (12)

Case 1: When $c_1 = c_2 = c_3 = c_4 = 0$, Manafian and Allahverdiyeva [34]

By considering $c_1 = c_2 = c_3 = c_4 = 0$
$(0, 0, 0, 0)$ is a steady state solution, Shahriari, et al. [35].

Case 2: When only $c_1 = 0$

Putting $c_1 = 0$ *in* (10), *we have*
$b_2c_2 + r_5c_3 + r_6c_4 = a_2$ (13)

Putting $c_1 = 0$ in (11), we get

$$b_3c_3 + r_8c_2 + r_9c_4 = a_3 \tag{14}$$

Putting $c_1 = 0$ in (12), we have

$$b_4c_4 + r_{11}c_2 + r_{12}c_3 = a_4 \tag{15}$$

$$\Rightarrow r_{11}c_2 = a_4 - b_4c_4 - r_{12}c_3$$

$$\Rightarrow c_2 = \frac{1}{r_{11}}(a_4 - b_4c_4 - r_{12}c_3) \tag{16}$$

Putting (16) in (13), we get

$$\frac{b_2}{r_{11}}(a_4 - b_4c_4 - r_{12}c_3) + r_5c_3 + r_6c_4 = a_2.$$

This implies

$$(r_{11}r_5 - r_{12}b_2)c_3 + (r_{11}r_6 - b_2b_4)c_4 = r_{11}a_2 - a_4b_2\,. \tag{17}$$

Putting (16) in (14), we have

$$b_3c_3 + \frac{r_8}{r_{11}}(a_4 - b_4c_4 - r_{12}c_3) + r_9c_4 = a_3 \quad \Longrightarrow \quad c_3 = \frac{r_{11}a_3 - r_8a_4 - (r_{11}r_9 - r_8b_4)c_4}{r_{11}b_3 - r_{12}r_8}. \tag{18}$$

Substituting (18) into (17), we get

$$\frac{(r_{11}r_5 - r_{12}b_2)(r_{11}a_3 - r_8a_4) - (r_{11}r_5 - r_{12}b_2)(r_{11}r_9 - r_8b_4)c_4}{r_{11}b_3 - r_{12}r_8} + (r_{11}r_6 - b_2b_4)c_4 = r_{11}a_2 - a_4b_2.$$

This implies

$$c_4 = \frac{(r_{11}b_3 - r_{12}r_8)(r_{11}a_2 - a_4b_2) - (r_{11}r_5 - r_{12}b_2)(r_{11}a_3 - r_8a_4)}{(r_{11}b_3 - r_{12}r_8)(r_{11}r_6 - b_2b_4) - (r_{11}r_5 - r_{12}b_2)(r_{11}r_9 - r_8b_4)} \tag{19}$$

Putting (19) into (17) gives

$$c_3 = \frac{-(r_{11}r_9 - r_8b_4)[(r_{11}b_3 - r_{12}r_8)(r_{11}a_2 - a_4b_2) - (r_{11}r_5 - r_{12}b_2)(r_{11}a_3 - r_8a_4)]}{(r_{11}b_3 - r_{12}r_8)[(r_{11}b_3 - r_{12}r_8)(r_{11}r_6 - b_2b_4) - (r_{11}r_5 - r_{12}b_2)(r_{11}r_9 - r_8b_4)]}.$$

Putting (19) and (18) in (15) yields

$$c_2 = \frac{a_4}{r_{11}} - \frac{b_4}{r_{11}}\left[\frac{(r_{11}b_3 - r_{12}r_8)(r_{11}a_2 - a_4b_2) - (r_{11}r_5 - r_{12}b_2)(r_{11}a_3 - r_8a_4)}{(r_{11}b_3 - r_{12}r_8)(r_{11}r_6 - b_2b_4) - (r_{11}r_5 - r_{12}b_2)(r_{11}r_9 - r_8b_4)}\right]$$

$$-\frac{r_{12}}{r_{11}}\left\{\frac{(r_{11}a_3 - r_8a_4)[(r_{11}b_3 - r_{12}r_8)(r_{11}r_6 - b_2b_4) - (r_{11}r_5 - r_{12}b_2)(r_{11}r_9 - r_8b_4)]}{-\frac{(r_{11}r_9 - r_8b_4)[(r_{11}b_3 - r_{12}r_8)(r_{11}a_2 - a_4b_2) - (r_{11}r_5 - r_{12}b_2)(r_{11}a_3 - r_8a_4)]}{(r_{11}b_3 - r_{12}r_8[(r_{11}b_3 - r_{12}r_8)(r_{11}r_6 - b_2b_4) - (r_{11}r_5 - r_{12}b_2)(r_{11}r_9 - r_8b_4)])}}\right\}$$

Let the values of C_2, C_3 and C_4 be A, B, and D respectively. Thus, (0, A, B, D) is a steady state solution, Dehghan [36].

Case 3: When only $c_2 = 0$

Putting $c_2 = 0$ in (9) gives

$$b_1c_1 + r_2c_3 + r_3c_4 = a_1 \tag{20}$$

Putting $c_2 = 0$ in (10) gives

$$r_4c_1 + r_5c_3 + r_6c_4 = a_2 \tag{21}$$

Putting $c_2 = 0$ in (11) gives

$$b_3c_3 + r_7c_1 + r_9c_4 = a_3 \quad \Rightarrow \quad c_1 = \frac{1}{r_7}(c_3 - b_3c_3 - r_9c_4). \tag{22}$$

Putting (22) into (20) gives

$$\frac{b_1}{r_7}(a_3 - b_3c_3 - r_9c_4) + r_2c_3 + r_3c_4 = a_1 \quad \Rightarrow \quad (r_2r_7 - b_1b_3)c_3 + (r_3r_7 - r_9b_1)c_4 = r_7a_1 - a_3b_1 \tag{23}$$

Putting (22) in (21) gives

$$\frac{r_4}{r_7}(a_3 - b_3c_3 - r_9c_4) + r_5c_3 + r_6c_4 = a_2 \quad \Rightarrow \quad c_3 = \frac{r_7a_2 - r_4a_3 - (r_6r_7 - r_4r_9)c_4}{r_5r_7 - r_4b_3} \tag{24}$$

Putting (24) in (23) gives

$$\frac{(r_2r_7 - b_1b_3)(r_7a_2 - r_4a_3) - (r_2r_7 - b_1b_3)(r_6r_7 - r_4r_9)}{r_5r_7 - r_4b_3}c_4 + (r_3r_7 - r_9b_1)c_4 = r_7a_1 - a_3b_1.$$

This implies

$$c_4 = \frac{(r_5r_7 - r_4b_3)(r_7a_1 - a_3b_1) - (r_2r_7 - b_1b_3)(r_7a_2 - r_4a_3)}{(r_5r_7 - r_4b_3)(r_3r_7 - r_9b_1) - (r_2r_7 - b_1b_3)(r_6r_7 - r_4r_9)} \tag{24}$$

Putting (25) in (24) gives

$$c_3 = \frac{-(r_6r_7 - r_4r_9)[(r_5r_7 - r_4b_3)(r_7a_1 - a_3b_1) - (r_2r_7 - b_1b_3)(r_7a_2 - r_4a_3)]}{(r_5r_7 - r_4b_3)[(r_5r_7 - r_4b_3)(r_3r_7 - r_9b_1) - (r_2r_7 - b_1b_3)(r_6r_7 - r_4r_9)]} \tag{25}$$

Putting (26) and (25) in (22), we have

$$\begin{aligned} c_1 = \frac{a_3}{r_7} - \frac{b_3}{r_7}&\left\{\frac{\begin{array}{c}(r_7a_2 - r_4a_3)[(r_5r_7 - r_4b_3)(r_3r_7 - r_9b_1) - (r_2r_7 - b_1b_3)(r_6r_7 - r_4r_9)] \\ -(r_6r_7 - r_4r_9)[(r_5r_7 - r_4b_3)(r_7a_1 - a_3b_1) - (r_2r_7 - b_1b_3)(r_7a_2 - r_4a_3)]\end{array}}{(r_5r_7 - r_4b_3)[(r_5r_7 - r_4b_3)(r_3r_7 - r_9b_1) - (r_2r_7 - b_1b_3)(r_6r_7 - r_4r_9)]}\right\} \\ &- \frac{r_9}{r_7}\left[\frac{(r_5r_7 - r_4b_3)(r_7a_1 - a_3b_1) - (r_2r_7 - b_1b_3)(r_7a_2 - r_4a_3)}{(r_5r_7 - r_4b_3)(r_3r_7 - r_9b_1) - (r_2r_7 - b_1b_3)(r_6r_7 - r_4r_9)}\right] \end{aligned} \tag{26}$$

Let the values of C_1, C_3 and C_4 be E, B, and D, respectively. Hence, (E, 0, B_1, D_1) is a steady state solution.

Case 4: When only C_3 =0

Putting C_3=0 in (9), we have

$$b_1c_1 + r_1c_2 + r_3c_4 = a_1 \tag{27}$$

Putting $c_3 = 0$ in (10) gives

$$b_2c_2 + r_4c_1 + r_6c_4 = a_2 \tag{28}$$

Putting $c_3 = 0$ in (12) gives

$$b_4c_4 + r_{10}c_1 + r_{11}c_2 = a_4 \quad \Rightarrow \quad c_2 = \frac{1}{r_{11}}(a_4 - b_4c_4 - r_{10}c_1) \tag{29}$$

Putting (29) in (27) gives

$$c_1 = \frac{r_{11}a_1 - r_1a_4 - (r_3r_{11} - r_1b_4)c_4}{r_{11}b_1 - r_1r_{10}} \tag{30}$$

Putting (29) in (28) gives

$$\frac{b_2}{r_{11}}(a_4 - b_4c_4 - r_{10}c_1) + r_4c_1 + r_6c_4 = a_2.$$

This implies

$$(r_4r_{11} - r_{10}b_2)c_1 + (r_6r_{11} - b_2b_4)c_4 = r_{11}a_2 - a_4b_2 \tag{31}$$

Putting (30) m (31) gives

$$c_4 = \frac{(r_{11}b_1 - r_1r_{10})(r_{11}a_2 - a_4b_2) - (r_4r_{11} - r_{10}b_2)(r_{11}a_1 - r_1a_4)}{(r_{11}b_1 - r_1r_{10})(r_6r_{11} - b_2b_4) - (r_4r_{11} - r_{10}b_2)(r_3r_{11} - r_1b_4)} \tag{32}$$

Putting (32) in (30) gives

$$c_1 = \frac{-(r_3r_{11} - r_1a_4)[(r_{11}b_1 - r_1r_{10})(r_6r_{11} - b_2b_4) - (r_4r_{11} - r_{10}b_2)(r_{11}a_1 - r_1a_4)]}{(r_{11}b_1 - r_1r_{10})[(r_{11}b_1 - r_1r_{10})(r_6r_{11} - b_2b_4) - (r_4r_{11} - r_{10}b_2)(r_3r_{11} - r_1b_4)]} \tag{33}$$

Putting (32) and (33) m (29), we get

$$c_2 = \frac{a_4}{r_{11}} - \frac{b_4}{r_{11}}\left[\frac{-(r_{11}b_1 - r_1r_{10})(r_{11}a_2 - a_4b_2) - (r_4r_{11} - r_{10}b_2)(r_{11}a_1 - r_1a_4)}{(r_{11}b_1 - r_1r_{10})(r_6r_{11} - b_2b_4) - (r_4r_{11} - r_{10}b_2)(r_3r_{11} - r_1b_4)}\right]$$
$$\frac{r_{10}}{r_{11}}\left\{\frac{(r_{11}a_1 - r_1a_4)[(r_{11}b_1 - r_1r_{10})(r_6r_{11} - b_2b_4) - (r_4r_{11} - r_{10}b_2)(r_3r_{11} - r_1b_4)] \\ -(r_3r_{11} - r_1b_4)[(r_{11}b_1 - r_1r_{10})(r_{11}a_2 - a_4b_2) - (r_4r_{11} - r_{10}b_2)(r_{11}a_1 - r_1a_4)]}{(r_{11}b_1 - r_1r_{10})[(r_{11}b_1 - r_1r_{10})(r_6r_{11} - b_2b_4) - (r_4r_{11} - r_{10}b_2)(r_3r_{11} - r_1b_4)]}\right\}$$

Let the values of c_1, c_2 *and* c_4 *be* E_1, A_1 and D_2 *respectively. Thus*, $(E_1, A_1, 0, D_2)$ is a steady state solution.

Case 5: when only $C_4 = 0$, Manafian [37]

Putting $C_4 = O$ in (9) gives

$$b_1c_1 + r_1c_3 = a_1 \tag{34}$$

Putting $c_4 = O$ *in* (10) gives

$$b_2c_2 + r_4c_1 + r_5c_3 = a_2 \tag{34}$$

Putting $c_4 = O$ *in* (11) gives

$$b_3c_3 + r_7c_1 + r_8c_2 = a_3 \quad \Rightarrow \quad c_1 = \frac{1}{r_7}(a_3 - b_3c_3 - r_8c_2) \tag{36}$$

Putting (36) in (34) give

$$\frac{b_1}{r_7}(a_3 - b_3c_3 - r_8c_2) + r_1c_2 + r_2c_3 = a_1 \Rightarrow (r_1r_7 - r_8b_1)c_2 + (r_2r_7 - b_1b_3)c_3 = r_7a_1 - a_3b_1 \tag{37}$$

Putting (36) in (35) gives

$$b_2c_2 + \frac{r_4}{r_7}(a_3 - b_3c_3 - r_8c_2) + r_5c_3 = r_7a_2 \quad \Longrightarrow \quad c_2 = \frac{r_7a_2 - r_4a_3 - (r_5r_7 - r_4b_3)c_3}{r_7b_2 - r_4r_8} \tag{38}$$

Putting (38) in (37) gives

$$\frac{(r_2r_7 - r_8b_1)(r_7a_2 - r_4a_3) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)c_3}{r_7b_2 - r_4r_8} + (r_2r_7 - b_1b_3)c_3 = r_7a_1 - a_3b_1.$$

This implies

$$c_3 = \frac{(r_7b_2 - r_4r_8)(r_7a_1 - a_3b_1) - (r_1r_7 - r_8b_1) - (r_7a_2 - r_4a_3)}{(r_7b_2 - r_4r_8)(r_2r_7 - b_1b_3) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)}. \tag{39}$$

Putting (39) in (38)

$$c_2 = \frac{r_7a_2 - r_4a_3 - (r_5r_7 - r_4b_3)}{r_7b_2 - r_4r_8}\left(\frac{(r_7b_2 - r_4r_8)(r_7a_1 - a_3b_1) - (r_1r_7 - r_8b_1) - (r_7a_2 - r_4a_3)}{(r_7b_2 - r_4r_8)(r_2r_7 - b_1b_3) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)}\right) \tag{40}$$

Putting (39) and (40) in (36) gives

$$\begin{aligned} c_1 = \frac{a_3}{r_7} - \frac{b_3}{r_7}\left[\frac{(r_7b_2 - r_4r_8)(r_7a_1 - a_3b_1) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)}{(r_7b_2 - r_4r_8)(r_2r_7 - b_1b_3) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)}\right] \\ - \frac{r_8}{r_7}\left\{\frac{\begin{array}{c}(r_7a_2 - r_4a_3[(r_7b_2 - r_4r_8)(r_2r_7 - b_1b_3) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)] \\ -(r_5r_7 - r_4b_3)[(r_7b_2 - r_4r_8)(r_7a_1 - a_3b_1) - (r_1r_7 - r_8b_1) - (r_7a_2 - r_4a_3)]\end{array}}{r_7b_2 - r_4r_8[(r_7b_2 - r_4r_8)(r_2r_7 - b_1b_3) - (r_1r_7 - r_8b_1)(r_5r_7 - r_4b_3)]}\right\} \end{aligned} \tag{41}$$

Let the values of C_1, C_2 and C_3 be E_2, A_2 and B_2respectively. Hence, $(E_{2,}A_{2,}B_{2,}0)$ is a steady state solution.

Initial Conditions

With the initial condition precise values $c_1(0) = c_{10}>0$, $c_2(0) = c_{20}>0$, $c_3(0) = c_{30}>0$, $c_4(0) = c_{40}>0$ where:
$c_1(t)$ defines the quantity of crude oil that is produced over time in the first oil well.
$c_2(t)$ defines the quantity of crude oil that is produced over time in the second oil well.
$c_3(t)$ defines the quantity of crude oil that is produced over time in the third oil well.
$c_4(t)$ defines the quantity of crude oil that is produced over time in the fourth oil well.

a_1, a_2, a_3 and a_4 defines the growth parameter b_1, b_2, b_3 and b_4 defines the intra-competition coefficient r_1, r_2, r_3… r_{12} defines the inter-competition coefficient, Okposo, et al. [38].

For the purpose of this study, we have assumed the following parameter values

$a_1 = 0.0662$, $b_1 = 0.072$, $r_1 = 0.0018$, $r_2 = 0.0012$, $r_3 = 0.0012$,
$a_2 = 1.3548$, $b_2 = 1.4054$, $r_4 = 0.0014$, $r_5 = 0.0012$, $r_6 = 0.00094$,
$a_3 = 0.0410$, $b_3 = 0.0147$, $r_7 = 0.0018$, $r_8 = 0.0015$, $r_9 = 0.0012$,
$a_4 = 0.0802$, $b_4 = 0.0222$, $r_{10} = 0.00222$, $r_{11} = 0.0013$, $r_{12} = 0.00165$

For t =0, we have considered the initial boundary conditions as

$c_1(0) = 0.5$ barrels of crude oil
$c_2(0) = 0.25$ barrels of crude oil
$c_3(0) = 0.8$ barrels of crude oil
$c_4(0) = 0.4$ barrels of crude oil

3.3 Method of analysis

This proposed study will focus on the application of Runge-kutta numerical method of order 45, Apanapudor, et al. [39] otherwise called the ordinary differential equation of order 45 (0DE45) numerical method which is computationally more efficient than its counterparts namely ODE23, ODE 23 TB and ODE15s. On examination of the effect of the data precision index on the interacting Niger Delta crude oil production, numerically simulated data due to variation of the initial condition such as (0.5, 0.25, 0.8, 0.4), we found four (4) standard deviation values namely 0.0743, for the first crude oil production data, 0.1617 for the second crude oil production data, 0.1683 for the third crude oil production data and 0.3532 for the fourth crude oil production data. In the list of standard deviation variation, we found that the standard deviation value of 0.0743 classifies the first crude oil production data as the best in this scenario.

3.3.1 Mathematical preliminary 1

From the initial mathematical analysis and the numerical simulation of the above four (4) model equations which do not have a closed form solution as in a traditional first order separable differential equations, we have formed the following functional analysis variables to be fundamental to the understanding of crude oil well interaction data. Without loss of generality, the variance of a set of data is usually defined as a function of the mean, that is to say the sum of the squared deviations from the mean divided by one less than the sample size.

Variance (σ) = $\frac{\sum(x-\bar{x})^2}{N-1}$, where: x is each item value of the distribution

$\bar{x}$ is the mean of the distribution
N is total number of cases in the distribution i.e the sample size.

Okwonu, et al. [23] Okwonu, et al. [30].

The mean is just the average, the value that is the sum of all values, divided by the number.
Mean ($\bar{x}$) = $\frac{\sum x}{N}$.While, standard deviation is defined as the positive square root of the variance. In other words, it is a statistic that measures the dispersion of a data set relative to its mean.

Standard Deviation = $\sqrt{\frac{\sum(x-\bar{x})^2}{N-1}}$

The central ideology is that the smaller the standard deviation the better the data that is specified. Therefore, the hypothesis of data precision depends on the philosophy of the smallest standard deviation.

i. Column one time dependent data (where the time variable is also called the length of producing the quantity of Niger Delta crude oil in barrels) in the first Niger delta Crude oil well, provided the initial condition of the initial quantity of crude oil in the first well is well defined.
ii. Column two (2) time dependent data (where the time variable is also called the length of producing the quantity of Niger Delta crude oil in barrel(s) in the second Niger delta crude oil well, provided the initial condition of the initial quantity of crude oil in the second well is well defined.

Under the estimated data precision index (EDPI), the first value in the first column represent the mean of the first column data, the second value below represent the mean of the second column data while the third and fourth value below in the first column represents the mean of the third and fourth column data respectively.
In the same vain the second column values represents the variance of the first, second, third and fourth column data in that order and the third column values represents the standard deviation of the first, second, third and fourth column data respectively.

3.3.2 Mathematical preliminary 2

3.3.2.1 Properties of data set

If a_n is a sequence of real numbers, such that a_n converges to L written as $a_n \to L$; then the sequence is said to converge to a finite limit L, Izevbizua and Apanapudor (2019), Apanapudor, et al. [40] Jain and Ahuja (2013).

In other words, a sequence (x_n), real or complex is said to converge to a limit L. if for each ∈>0, ∋ a positive integral N (depending on ∈) such that $|x_n-1| <\in$ for all $n \geq N$ (Jain and Ajuja, 2013).

Consider the hypothetical data D_1 and D_2 as shown in Table 1 below:

Table 1. Some hypothetical Data Set D1 & D2

D_1	D_2	D_1+D_2	D_1*D_2	D_1/D_2
0.12	1.12	1.24	0.1344	1.1071
0.14	1.14	1.28	0.1596	0.1228
0.16	1.16	1.32	0.1856	0.1379
0.18	1.18	1.36	0.2124	0.1525
2.24	3.24	5.48	7.2576	0.6914
2.36	3.36	5.72	7.9296	0.7024
3.20	4.20	7.4	13.44	0.7619
4.48	5.48	9.96	24.5504	0.8175

How do we observe that the data has a limit?

To answer this question, if a sequence of real numbers (a_n), converges to a finite number L, we write it as $a_n \rightarrow L$ where a_j, j = 1,2, … ,n are called the elements of the sequence, provided the sequence of data obeys the law of monotone sequences. From the hypothetical data, the limiting value of the D_2 is 4.48 whereas the limiting value of the D_2 data is 5.48. Apanapudor and Otunta [41].

3.4 Bounded sequences

Following Apanapudor [41] Apanapudor and Aderibigbe (2018), Jain and Ahuja (2013), if a sequence a_n of a real number converges to a finite limit L, that is $a_n \rightarrow L_1$ then the sequence is said to be bounded. In other words, a sequence is bounded if it is bounded above and below, that is, if there is a number K, less than or equal to all the terms of the sequence and another number K greater than or equal to all the terms of the sequence

3.5 Sum of two bonded sequences

Similarly, if $a_n \rightarrow L_1$ and $a_m \rightarrow L_2$ be two bounded sequences, then the sequence $a_n + a_m$ converges to $L_1 + L_2$ shown in column three in Table 1. Jain and Ahuja (2013), Olaosebikan, et al. [42] Mamadu and Apanapudor (2017). For example, a_n of D_1 data converges to 4.48 ($a_n \rightarrow 4.48$) and a_m of D_2 data converges to 5.48 ($\rightarrow 5.48$), therefore $a_n + a_m \rightarrow L_1 + L_2 = 4.48 + 5.48 = 9.96$, hence the sum of two bonded sequence are bounded.

3.6 Product of two bounded sequences

Following Jain and Ahuja (2013), Okwonu, et al. (2021a),Okwonu, et al. (2020), mathematically if a_n and a_m are sequence of real number such that $a_n * a_m \rightarrow L_1 * L_2$ of real number is said to be bounded. For example, a_n of D_1 data converges to 4.48 ($a_n \rightarrow 4.48$) and a_m of D_2 data converges to 5.48 ($a_m \rightarrow 5.48$), hence their product $a_n * a_m \rightarrow L_1 * L_2 = 4.48 \text{ x } 3.48 = 24.5304$ (Column 4).

3.7 Ratio of two bounded sequences

Following Jain and Ahuja (2013), Iweobodo, et al. [43] Omokoh, et al. [44] mathematically, if a_n and $a_m$are sequences of real numbers such that $a_n \rightarrow L_1$ and $a_m \rightarrow L_2$ then the ratio of sequences of real numbers $a_n/a_m \rightarrow L_1/L_2$ is said to be bounded provided L_2 is not equal to zero ($L_2=0$).

For example a_n of D_1 data converges to 4.48 ($a_n \rightarrow 4.48$) and a_m of D_2 data converges to 5.58 ($a_m \rightarrow 5.48$) then $a_n/a_m \rightarrow L_1/L_2 = 4.48/5.48 = 0.8175$ (column 5).

3.8 Monotone sequence

Following Jain and Ahuja (2013), Apanapudor, et al. (2023a) if a_n is a sequence of real numbers such that for all n satisfying $a_{n+1} > a_n$; then the sequence a_n is said to be increasing monotonically. On the other hand, if a_{n+1} is strictly less than a_ni.e a_{n+1}, $< a_n$ for all n, then the sequence of real number a_n is said to be monotone, if it is both increasing monotonically and decreasing monotonically.

In summary, if a_n and a_m are sequence of real numbers such that $a_n \rightarrow L_1$ and $a_m \rightarrow L_2$ then the sequence a_m and a_m which converges to the finite limit L_1 and L_2 are said to be bounded, if they are bounded then the following properties will be satisfied, Okwonu, et al. (2021a), Okwonu, et al. [45] Apanapudor and Olowu [46].

i. $a_n + a_m \rightarrow L_1 + L_2$ (sum)
ii. $a_n . a_m \rightarrow L_1 . L_2$ (product)
iii. $a_n / a_m \rightarrow L_1 / L$ Provided $L_2 \neq 0$ (ratio)

For the purpose of this study, data sets that satisfy the properties of a bounded sequence can be tested for the smallest standard deviation, otherwise called the data precision index.

4 Numerical Results

The following tables present the numerical computations emanating from the data so far collected.

Table 2. Computing the data precision of interacting Niger Delta crude oil data due to the initial condition (0.7 0.35 1.0 0.50) using ODE45 numerical methods

D1	D2	D3	D4
0.7000	0.3500	1.0000	0.5000
0.7090	0.6624	1.0238	0.5335
0.7172	0.8635	1.0472	0.5686
0.7249	0.9387	1.0706	0.6054
0.7322	0.9576	1.0940	0.6439
0.7391	0.9615	1.1174	0.6842
0.7456	0.9624	1.1409	0.7264
0.7518	0.9614	1.1644	0.7704
0.7577	0.9616	1.1878	0.8162
0.7632	0.9627	1.2113	0.8637
0.7684	0.9624	1.2347	0.9131
0.7733	0.9609	1.2580	0.9641
0.7779	0.9612	1.2813	1.0167
0.7822	0.9620	1.3044	1.0709
0.7862	0.9612	1.3274	1.1266
0.7899	0.9601	1.3503	1.1836
Estimated data precision index with respect to the crude oil production data			
0.7934	0.9612	1.3729	1.2418
0.7966	0.9631	1.3954	1.3012
0.7996	0.9618	1.4177	1.3615
0.8023	0.9596	1.4398	1.4226
0.8048	0.9609	1.4616	1.4844
0.8071	0.9617	1.4832	1.5467
0.8092	0.9604	1.5045	1.6093
0.8111	0.9602	1.5255	1.6721
0.8128	0.9607	1.5462	1.7348
Estimated data precision index with respect to the third crude oil production data			
0.7702	0.0012	0.0346	
0.9200	0.0179	0.1339	
1.2784	0.0284	0.1685	
1.0545	0.1484	0.3852	

Table 3. Computing the data precision of four interacting Niger Delta crude oil data due to the initial condition (0.8 0.4 1.1 0.55) using ODE45 numerical methods

0.8000	0.4000	1.1000	0.5500
0.8043	0.7055	1.1241	0.5859
0.8081	0.8805	1.1479	0.6235
0.8114	0.9425	1.1715	0.6627
0.8144	0.9585	1.1950	0.7037
0.8172	0.9624	1.2185	0.7465
0.8198	0.9616	1.2420	0.7912
0.8221	0.9615	1.2654	0.8376
0.8243	0.9615	1.2887	0.8857
0.8263	0.9615	1.3120	0.9356
0.8281	0.9616	1.3351	0.9871
0.8297	0.9614	1.3581	1.0402
0.8311	0.9610	1.3809	1.0948
0.8324	0.9609	1.4035	1.1508
0.8336	0.9616	1.4259	1.2082
0.8346	0.9623	1.4481	1.2667
0.8355	0.9614	1.4701	1.3262
0.8362	0.9603	1.4918	1.3866
0.8368	0.9610	1.5132	1.4478
0.8373	0.9613	1.5343	1.5095
0.8377	0.9602	1.5552	1.5717
0.8380	0.9596	1.5757	1.6341
0.8382	0.9608	1.5959	1.6966
0.8383	0.9613	1.6158	1.7590
0.8383	0.9607	1.6353	1.8210

Estimated data precision index with respect to the fourth crude oil production data

0.8269	0.0001	**0.0118**
0.9244	0.0147	0.1214
1.3762	0.0273	0.1652
1.1289	0.1576	0.3969

Table 4. Computing the data precision of four (4) interacting Niger Delta crude oil data due to the initial condition (0.9 0.45 1.2 0.6) using ODE45 numerical methods

0.9000	0.4500	1.2000	0.6000
0.8984	0.7430	1.2241	0.6382
0.8964	0.8974	1.2478	0.6780
0.8944	0.9462	1.2713	0.7195
0.8923	0.9582	1.2947	0.7628
0.8904	0.9613	1.3180	0.8079
0.8884	0.9614	1.3411	0.8547
0.8866	0.9614	1.3641	0.9033
0.8847	0.9614	1.3870	0.9535
0.8830	0.9613	1.4097	1.0053
0.8812	0.9611	1.4322	1.0588
0.8795	0.9611	1.4545	1.1136
0.8778	0.9613	1.4766	1.1698
0.8761	0.9614	1.4984	1.2273
0.8745	0.9607	1.5200	1.2859
0.8729	0.9600	1.5413	1.3455
0.8713	0.9606	1.5623	1.4059
0.8697	0.9616	1.5830	1.4670
0.8681	0.9608	1.6033	1.5287
0.8666	0.9600	1.6233	1.5907

0.8651	0.9610	1.6430	1.6529
0.8635	0.9620	1.6623	1.7151
0.8620	0.9605	1.6813	1.7771
0.8605	0.9586	1.6999	1.8388
0.8590	0.9597	1.7180	1.9000
Estimated data precision index with respect to the fifth crude oil production data			
0.8785	0.0002	**0.0126**	
0.9285	0.0119	0.1093	
1.4703	0.0255	0.1598	
1.2000	0.1651	0.4064	

Table 5. Computing the data precision of four (4) interacting Niger Delta crude oil data due to the initial condition (1.0 0.5 1.3 0.65) using ODE45 numerical methods

1.0000	0.5000	1.3000	0.6500
0.9910	0.7761	1.3238	0.6903
0.9823	0.9094	1.3471	0.7322
0.9741	0.9501	1.3702	0.7758
0.9663	0.9602	1.3931	0.8212
0.9591	0.9614	1.4158	0.8683
0.9523	0.9612	1.4383	0.9171
0.9459	0.9612	1.4606	0.9675
0.9399	0.9612	1.4828	1.0195
0.9342	0.9610	1.5047	1.0731
0.9289	0.9608	1.5263	1.1281
0.9238	0.9610	1.5477	1.1844
0.9190	0.9614	1.5688	1.2419
0.9145	0.9613	1.5896	1.3005
0.9102	0.9600	1.6100	1.3601
0.9060	0.9593	1.6302	1.4204
0.9021	0.9607	1.6500	1.4814
0.8984	0.9613	1.6695	1.5429
0.8948	0.9603	1.6886	1.6047
0.8914	0.9605	1.7073	1.6666
0.8881	0.9614	1.7256	1.7286
0.8850	0.9612	1.7436	1.7903
0.8819	0.9598	1.7612	1.8516
0.8790	0.9600	1.7783	1.9124
0.8762	0.9604	1.7951	1.9725
Estimated data precision index with respect to the sixth crude oil production data			
0.9258	0.0014	0.0373	
0.9325	0.0095	0.0977	
1.5611	0.0233	0.1527	
1.2681	0.1712	0.4137	

From Table 1, we observed that the four (4) interacting data are bounded sequences in which the first column data converges to a finite limit value of 0.7899,; the second column data converges to a finite limit value of 0.9601, the third to 1.3503 and the fourth 1.1836. The corresponding ODE 45 computed data precision indices of the four(4) interacting data set are (1) 0.0346, (ii) 0.1339 (iii) 0.1685 (iv)0.3852 with the smallest value as 0.0346 aligning with the first column [47,48].

A look at Table 2, we observed that the four (4) interacting data are bounded sequences in which the first column data converges to a finite limit value of 0.8383, the second column data converges to a finite limit value of 0.9607, the third column data converges to a finite limit value of 1.6353 and the fourth column data converges to a finite limit value of 1.8210 provided the specified initial condition is defined by (0.8, 0.4, 1.1, 0.55).

The corresponding ODE 45 computed data precision indices of the fourth (4) interacting data are 0.0118 for the first column data, 0.1214 for the second column, 0.1652, for the third column data and 0.3969 for the fourth column data in which the smallest data precision index value of 0.0118 aligns with the first column data [49,50].

Again in Table 3, the four (4) interacting data set are bounded sequences in which the first column data converges to a finite limit value of 0.8590, the second column data converges to a finite limit value of 0.9597, the third column data converges to a finite limit value of 1.7180 and the fourth column data converges to a finite limit value of 1.900 provided the specified initial condition is define by (0.9, 0.45, 0.12, 0.6).

The corresponding ODE 45 computed data precision indices of the four (4) interacting data are 0.0126 for the first column data, 0.1093 for the second column data, 0.1598 for the third column data and 0.4064 for the fourth column data in which the smallest data precision index value of 0.0126 aligns with the first column data [51].

Also in Table 4, the four (4) interacting data set are also bounded sequences in which the first column data converges to a finite limit value of 0.8762, the second column data converges to a finite limit values of 0.9604, the third column data converges to a finite limit value of 1.7951 and the fourth column data converges to a finite limit value of 1.9725 provided the specified initial condition is defined by (1.0, 0.5, 1.3, 0.65).

The corresponding ODE 45 computed data precisions indices of the four (4) interacting data are 0.0373 for the first column data, 0.0977 for the second column data, 0.1527 for the third column data and 0.4137 for the fourth column data in which the smallest data precision index value of 0.0373 aligns with the first column data.

5 Conclusion

This study have significantly applied the continuous dynamical system of a non-linear first order differential equation to study the effect of data precision index on the Niger Delta crude oil production data numerically. With a variation in the initial condition, we have obtained the data precision index of 0.0346 for the first column data, 0.0118 for the second column data,0.0126 for the third column data and 0.0373 for the fourth column data as shown in the bold font in Tables 1, 2, 3, and 4 respectively, which represent oil well one (W1), oil well two (W2), oil well three (W3) and oil well four (W4); and have concluded that 0.0118(IC15) and 0.0126(IC17) are the best fit-data with a precision index of 0.0126. This present numerical approach of proving a fundamental functional analysis result and research done within the mathematical analysis, extended to other types of data sets such as fossil fuel emission in the atmosphere, biodiversity quantification, geoscience, relative humidity and stock exchange numerically simulated data and as well as in the ecosphere assess prediction to mention but a few. In particular, the production of crude oil data, the present data precision analysis can provide a further insight which is rarely done for a better understanding of the revenue derived from crude oil production in the Niger Delta region of Nigeria. Therefore, there should be consistent use of a single numerical method to avoid conflicting prediction of data precision value. The choice of the initial boundary conditions should agree with the dimension, in other to obtain the expected prediction of crude oil production.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

Competing Interests

Authors have declared that no competing interests exist.

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