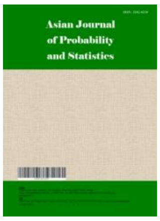

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The Iwok-Nwikpe Distribution: Statistical Properties and Its Application

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, a new continuous probability distribution named Iwok-Nwikpe distribution is proposed. Some essential statistical properties of the proposed probability distribution have been derived. The graphs of the survival function, probability density function (p.d.f) and cumulative distribution function (c.d.f) were plotted at different values of the parameter. The mathematical expression for the moment generating function (mgf) was derived. Consequently, the first three crude moments were obtained; the distribution of order statistics, the second and third moments corrected for the mean have also been derived. The parameter of the Iwok-Nwikpe distribution was estimated by means of maximum likelihood technique. To establish the goodness of fit of the Iwok-Nwikpe distribution, three real data sets from engineering and medical science were fitted to the distribution. Findings of the study revealed that the Iwok-Nwikpe distribution performed better than the one parameter exponential distribution and other competing models used for the study.

Keywords: Parameter estimation; cumulative distribution function; order statistics; exponential distribution; survival function and goodness of fit.

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1 Introduction

Latest studies in probability distributions gives evidence that most standard or conventional probability distributions do not satisfactorily fit some real life data sets gotten from areas of study such as medical science, engineering, biological sciences and so forth. As a result of this underperformance, a lot of efforts have been geared towards improving the flexibility of the classical distribution. Consequently, several techniques now exist for improving the flexibility of existing probability distributions. Some transformations that enhance the performance or the flexibility of classical distributions have also been proposed by statisticians recently. An archetypal case in point of such transformations is the generalized family of distributions amongst which we have the transmuted family of distributions developed by Shaw and Buckley [1], Exponentiated Generalised (EG) family of distributions by Cordeiro et al. [2] amongst others. The flexibility of a lot of classical distributions has been enhanced using the approach of generalization. For example, Ghitany [3] transformed the Pareto type 1 distribution by the use of the Marshall Olkin family of distributions by [4] and it was proven that the transformed distribution outperformed the baseline distribution for some data sets. Also, Krishna [5] modified the uniform distribution using theMarshall generalized family. The generalized uniform distribution was also found to be more flexible than the parent uniform distribution.

In recent studies, mixing two or more classical distributions has also gained prominence. Andrazej & Vladik [6] has shown that two or more distribution could be added to form a new distribution. It has been proven that new flexible probability distributions could be efficiently generated using the mixture approach; as a result, some new probabilistic models have been derived. Using the mixture technique, Nwikpe and Essi [7] derived the TPAN distribution, Lindley [8] proposed the Lindley distribution which is a mixture of exponential distribution with parameter θ and gamma $(2,\theta)$ with mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$. The probability density function (p.d.f) and cumulative distribution function (c.d.f) of the Lindley distribution are given below:

$$f(x\,;\,\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}\,;\,x>0,\theta>0 \tag{1}$$

$$F(x\,;\,\theta)=1-\frac{\theta+1+\theta x}{\theta+1}(1+x)e^{-\theta x}\,;\,x>0,\theta>0 \tag{2}$$

Shanker [9] derived a new distribution called the Shanker distribution using the same mixture approach. The baseline distribution of the Shanker distribution are the exponential and gamma distributions. The Shanker's p.d.f and c.d.f were:

$$f(x\,;\,\theta)=\frac{\theta^2}{\theta+1}(\theta+x)e^{-\theta x}\,;\,x>0,\theta>0 \tag{3}$$

$$F(x\,;\,\theta)=1-\frac{(\theta^2+1)+\theta x}{\theta^2+1}e^{-\theta x}\,;\,x>0,\theta>0 \tag{4}$$

respectively with the mixing ratios $\frac{\theta^2}{\theta^2+1}$ and $\frac{\theta^2}{\theta^2+1}$. Using some data sets, it was established that the Shanker distribution was more flexible than its baseline distribution.

The Sujatha distribution proposed by Shanker [10] is a three-components mixed model. The parent distributions of the Sujatha distributions are gamma (2, θ), exponential(θ) and gamma(3, θ). Shanker [10] obtained the p.d.f. and c.d.f. of the Sujatha distribution as:

$$f(x\,;\,\theta)=\frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x}\,;\,x>0,\theta>0 \tag{5}$$

$$F(x\,;\,\theta)=1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x}\,;\,x>0,\theta>0 \tag{6}$$

The Sujatha distribution was found to have a good fit with increasing hazard rate than the Shanker, exponential, and Lindley distribution. Shanker [10] proposed the Amarendra distribution of the form:

$$f(x\,;\,\theta) = \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}(1+x+x^2+x^3)e^{-\theta x}\,;\; x>0, \theta>0 \tag{7}$$

The Amarendra distribution is a four components mixture of gamma and exponential distribution with mixing proportions:

$$\frac{\theta^3}{\theta^3+\theta^2+2\theta+6}, \frac{\theta^2}{\theta^3+\theta^2+2\theta+6}, \frac{2\theta}{\theta^3+\theta^2+2\theta+6} \text{ and } \frac{6}{\theta^3+\theta^2+2\theta+6}.$$

Shanker [10] found that the Amarendra distribution have monotonically increasing hazard rate for modeling lifetime data and performed better than the aforementioned distributions.

In the present study, we derive a new probability density function with a simpler structure than the Amarendra and Sujatha type; and called it 'Iwok-Nwikpe distribution'. We also look at some statistical properties and other relevant characteristics of this new distribution.

2 The Iwok-Nwikpe Distribution

Let X be a continuous random variable, the random variable X is said to follow the Iwok-Nwikpe (I-N) distribution if its density function (p.d.f.) is given by:

$$f(x\,;\,\phi) = \frac{\phi^3}{\phi+2}(x^2+x)e^{-\phi x}\,;\; x>0, \phi>0 \tag{8}$$

Where ϕ is the scale parameter.

The distribution in (8) is a mixture of gamma $(2, \phi)$ and gamma $(3, \phi)$ distributions.

In general, a random variable X is said to follow a gamma distribution with parameters α and λ if its p.d.f. is:

$$f(x;\,\alpha,\lambda) = \frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)}\;;\; \alpha>0, \lambda>0\,, x>0$$

The baseline p.d.f.s of the Iwok-Nwikpe are:

$$f_1(x\,;\,\phi) = \text{gamma}(2,\phi) = \frac{\phi^2 x}{\Gamma(2)}e^{-\phi x} \tag{9}$$

And

$$f_2(x\,;\,\phi) = \text{gamma}(3,\phi) = \frac{\phi^3 x^2}{\Gamma(3)}e^{-\phi x} \tag{10}$$

with the mixing proportions:

$$p_1 = \frac{\phi}{\phi+2} \text{ and } p_2 = \frac{2}{\phi+2}$$

so that (8) can be expressed as:

$$f(x\,;\,\phi) = f_1(x\,;\,\phi)p_1 + f_2(x\,;\,\phi)p_2 \tag{11}$$

3 The Graphs of the p.d.f. of the Iwok-Nwikpe Distribution

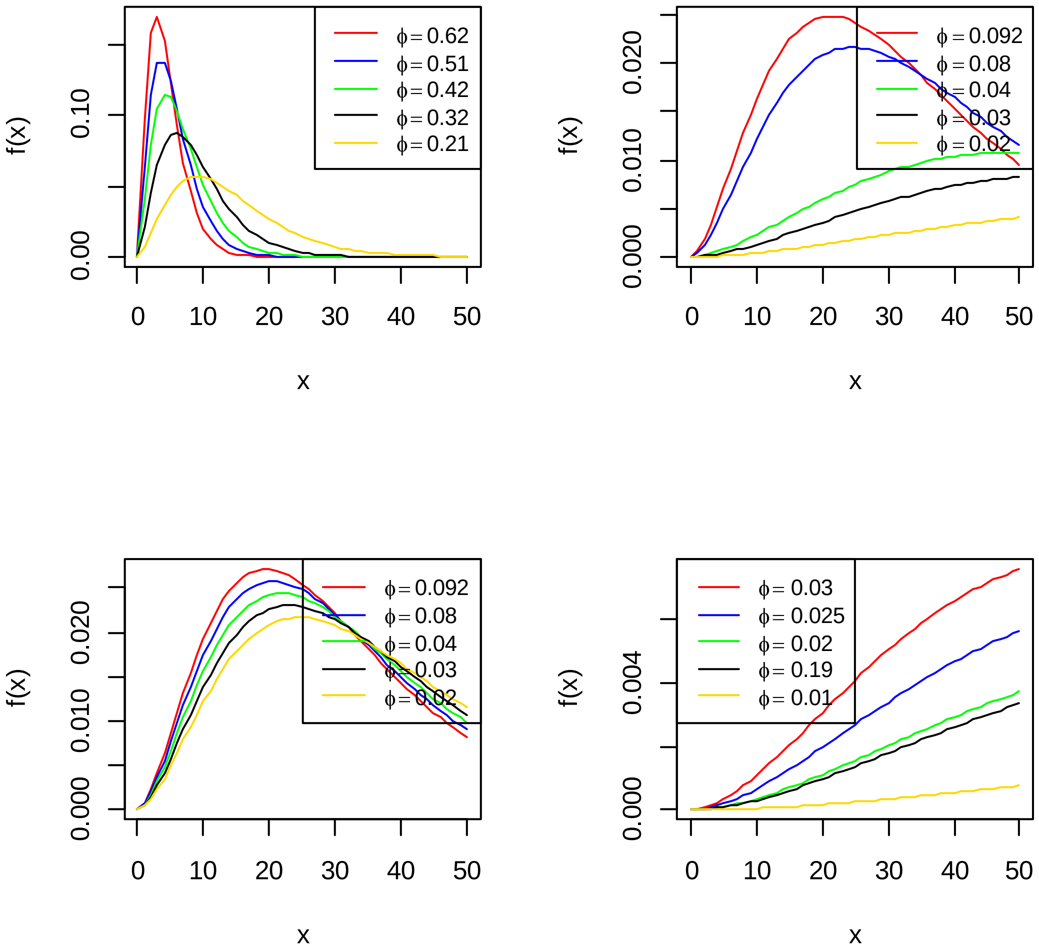

Fig. 1. Graphs of the p.d.f. of the I-N distribution for different values of ϕ

4 The Cumulative Distribution Function of the Iwok-Nwikpe Distribution

For any arbitrary random variable X, the cumulative distribution function(c.d.f.) is given as:

$$F(x) = \mathrm{P}(X \leq x), x \in \mathbb{R}$$

$$\Rightarrow \mathrm{P}(X \leq x) = \int_{0}^{x} f(t)dt\,; \text{ where } t \sim (\mathrm{I-N})$$

Hence for (I-N) distribution,

$$F(x\,;\,\phi) = \frac{\phi^3}{(\phi+2)} \int_{0}^{x} (t^2+t)e^{-\phi t}dt$$

$$= \frac{\phi^3}{(\phi+2)} \left[\frac{-e^{-\phi t}}{\phi}(t^2+t) - \frac{e^{-\phi t}(2t+1)}{\phi^2} - 2\frac{e^{-\phi t}}{\phi^3} \right]_0^x$$

$$= \left[1 - \left(1 + \frac{\phi^2 x(x+1) + 2\phi x}{\phi + 2}\right) \exp(-\phi x)\right] \qquad (12)$$

The graphs of the c.d.f. for different values of ϕ are as shown below:

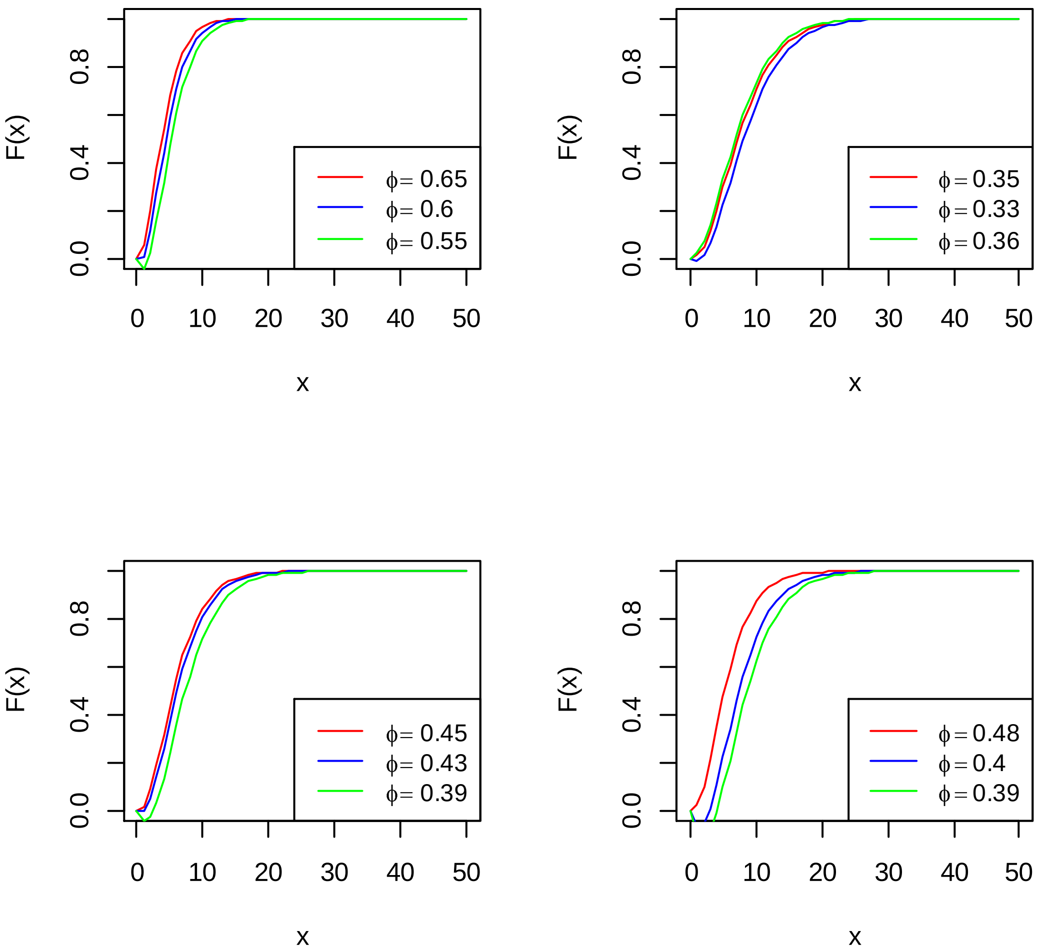

Fig. 2. Graphs of the c.d.f. of the I-N distribution for different values of ϕ

5 The Crude Moments of the I-N Distribution

The kth raw moment of a random variable X is given as follows:

$$\mu'_k = E[X^k] = \int_{-\infty}^{\infty} x^k f(x) dx$$

If $f(x)$ is the p.d.f. of the I-N distribution, then

$$\mu'_k = \frac{\phi^3}{\phi + 2} \int_0^{\infty} x^k (x^2 + x) e^{-\phi x} dx$$

$$= \frac{\phi^3}{\phi + 2} \int_0^{\infty} x^{k+2} e^{-\phi x}\, dx + \int_0^{\infty} x^{k+1} e^{-\phi x}\, dx$$

$$= \frac{\phi^3}{\phi + 2}\left(\frac{\Gamma(k+3)}{\phi^{k+3}} + \frac{\Gamma(k+2)}{\phi^{k+2}}\right)$$

$$= \frac{\phi^3}{\phi + 2}\left[\frac{\Gamma(k+3) + \phi\Gamma(k+2)}{\phi^{k+3}}\right]$$

$$\Rightarrow \mu'_k = \frac{(k+2)! + \phi(k+1)!}{\phi^k(\phi + 2)} \quad (13)$$

Thus, the first four uncorrected moments of the I-N distribution is given as follows:

$$\mu'_1 = \frac{6 + 2\phi}{\phi(\phi + 2)},\ \mu'_2 = \frac{24 + 6\phi}{\phi^2(\phi + 2)},\ \mu'_3 = \frac{120 + 24\phi}{\phi^3(\phi + 2)},\ \mu'_4 = \frac{720 + 120\phi}{\phi^4(\phi + 2)}$$

where $\frac{6+2\phi}{\phi(\phi + 2)}$ is the mean of the distribution.

6 The Second Central Moment (Variance) of the Iwok-Nwikpe Distribution

Using the relationship between the raw and central moments we obtain the variance or second central moment of the Iwok-Nwikpe as follows:

$$\mu_2 = \sigma^2 = E[X - \mu]^2 = E[X^2] - \{E[X]\}^2 = \mu'_2 - (\mu'_1)^2$$

$$= \frac{24 + 6\phi}{\phi^2(\phi + 2)} - \left[\frac{6 + 2\phi}{\phi(\phi + 2)}\right]^2 = \frac{10\phi^2 + 60\phi + 84}{\phi^2(\phi + 2)} \quad (14)$$

7 The Third Central Moment of the I-N Distribution

Recall that $\mu_3 = E[X - \mu]^3 = \mu'_3 - 3\mu\mu'_2 + 2\,\mu'_3$, using equations (13) and (14) the third central moment of the N-I distribution is obtained as follows:

$$\mu_3 = \frac{120 + 24\phi}{\phi^3(\phi + 2)} - 3\left[\frac{6 + 2\phi}{\phi(\phi + 2)}\right]\left[\frac{24 + 6\phi}{\phi^2(\phi + 2)}\right] + 2\left[\frac{120 + 24\phi}{\phi^3(\phi + 2)}\right]^2$$

$$= \frac{12[\phi^3 + 5\phi^2 + 48\phi + 22]}{\phi^3(\phi + 2)^3} \quad (15)$$

8 The Moment Generating Function (m.g.f) of the Iwok-Nwikpe Distribution

The m.g.f. of a random variable X which follows the I-N distribution is given by

$$M_X(t) = \frac{\phi^3}{\phi + 2}\int_0^\infty e^{xt}\,(x^2 + x)e^{-\phi x}dx$$

$$= \left(\frac{\phi^3}{\phi + 2}\right)\int_0^\infty x^2 e^{-x(\phi - t)}\,dx + \int_0^\infty x e^{-x(\phi - t)}\,dx$$

$$= \frac{\phi^3}{\phi + 2}\left[\frac{\Gamma(3)}{(\phi - t)^3} + \frac{\Gamma(2)}{(\phi - t)^2}\right]$$

$$= \frac{\phi^3}{\phi+2}\left[\frac{2}{(\phi-t)^3} + \frac{1}{(\phi-t)^2}\right]$$

$$= \left(\frac{\phi^3}{\phi+2}\right)\left\{\frac{2}{\phi^3}\sum_{k=0}^{\infty}\binom{k+2}{k}\left(\frac{t}{\phi}\right)^k + \frac{1}{\phi^2}\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\phi}\right)^k\right\}$$

$$= \sum_{k=0}^{\infty}\left[\frac{(k+2)(k+1)+\phi(k+1)}{\phi+2}\right]\left(\frac{t}{\phi}\right)^k \tag{16}$$

The rth raw moments are the coefficients of $\frac{t^r}{r!}$ in equation (16) above. Hence,

$$\mu_r' = \frac{r!\,[(r+2)(r+1)+\phi(r+1)]}{\phi^r(\phi+2)}$$

9 Distribution of Order Statistics

Assuming $X_1, X_2, \dots, X_n$ are independent continuous random variables from I-N distribution, each with p.d.f. $f(x)$ and c.d.f. $F(x)$; then $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$ The X_is arranged in order of increasing magnitude are the order statistics. The p.d.f. of the kth order statistic [$Y = X_{(k)}$] is given by:

$$f_Y(y) = \frac{n!}{(n-k)!\,(k-1)!}\sum_{j=0}^{n-k}\binom{n-k}{j}(-1)^j F^{k+j-1}(y)\,f(y)$$

Thus, for the I-N distribution:

$$f_Y(y) = \frac{n!}{(n-k)!\,(k-1)!}\sum_{j=0}^{n-k}\binom{n-k}{j}(-1)^j$$
$$\times\left\{\left[1-\left(1+\frac{\phi^2x(x+1)+2\phi x}{\phi+2}\right)\exp(-\phi x)\right]\right\}^{k+j-1}\frac{\phi^3}{\phi+2}(x^2+x)e^{-\phi x}$$

$$= \frac{n!\,\phi^3(x^2+x)e^{-\phi x}}{(n-k)!\,(k-1)!\,(\phi+2)}\sum_{j=0}^{n-k}\sum_{q=0}^{\infty}\binom{k+j+1}{q}\binom{n-k}{j}(-1)^{j+q}$$
$$\times\left\{\left[1+\frac{\phi^2x(x+1)+2\phi x}{\phi+2}\right]\exp(-\phi x)\right\}^q$$

Recall: $(1+x)^k = \sum_{i=0}^{k}\binom{k}{i}x^i$ and let $p = k+j+1$

$$\Rightarrow f_Y(y) = \frac{n!\,\phi^3(x^2+x)e^{-\phi x}}{(n-k)!\,(k-1)!\,(\phi+2)}\sum_{i=0}^{q}\sum_{j=0}^{n}\sum_{q=0}^{\infty}\binom{p}{q}\binom{n-k}{j}(-1)^{j+q}$$
$$\times\left[\frac{\phi^2x(x+1)+2\phi x}{\phi+2}\right]^i e^{-\phi xq}$$

$$\Rightarrow f_Y(y) = \frac{n!\,\phi^3}{(n-k)!\,(k-1)!\,(\phi+2)^{i+1}}\sum_{i=0}^{q}\sum_{j=0}^{n}\sum_{q=0}^{\infty}\binom{p}{q}\binom{n-k}{j}(-1)^{j+q}$$

$$\times (x^2 + x)e^{-\phi x(1+q)}[\phi^2 x(x+1) + 2\phi x]^i$$

10 Maximum likelihood Estimate of the parameter of the I-N Distribution

Let $x_1, x_2, x_3, \dots, x_n$ be a $n -$ dimensional random sample from the I-N Distribution. Let the likelihood function of n be L. By definition;

$$L = \prod_{i=1}^{n} f(x_1, x_2, \dots, x_n\,; \phi) = \prod_{i=1}^{n} f(x_i\,; \phi)$$

$$= \prod_{i=1}^{n} \frac{\phi^3}{\phi + 2}(x_i^2 + x_i)\, e^{-\phi x_i}$$

$$= \left[\frac{\phi^3}{\phi + 2}\right]^n \prod_{i=1}^{n}(x_i^2 + x_i)\, e^{-\phi \sum_{i=1}^{n} x_i}$$

$$\log L = n\log\left(\frac{\phi^3}{\phi + 2}\right) + \log\sum_{i=0}^{n}(x_i^2 + x_i) - \phi\sum_{i=1}^{n} x_i \qquad (17)$$

Equation (17) is the log of the likelihood function of the I-N distribution. The model parameter ϕ is estimated by differentiating (17) with respect to ϕ and set it equals to 0. Thus, we have:

$$\frac{\partial \log L}{\partial \phi} = \frac{n(2\phi + 6)}{\phi(\phi + 2)} - \sum_{i=1}^{n} x_i = 0$$

$$\Rightarrow \frac{n(2\phi + 6)}{\phi(\phi + 2)} - \sum_{i=1}^{n} x_i = 0 \qquad (18)$$

Equation (18) could be solved using R programming with a given data set. The solution to equation (18) gives the maximum likelihood estimate of ϕ.

11 Survival Function of the New I-N Distribution

Generally, the mathematical representation of the survival function of a random variable X is givenas:

$$s(x) = 1 - F(x)$$

For a random variable X which has the I-N distribution, its survivor function is given by:

$$s(x) = 1 - \left[1 - \left(1 + \frac{\phi^2 x(x+1) + 2\phi x}{\phi + 2}\right)\exp(-\phi x)\right]$$

12. The Graphs of the Survival Function of the Iwok-Nwikpe Distribution

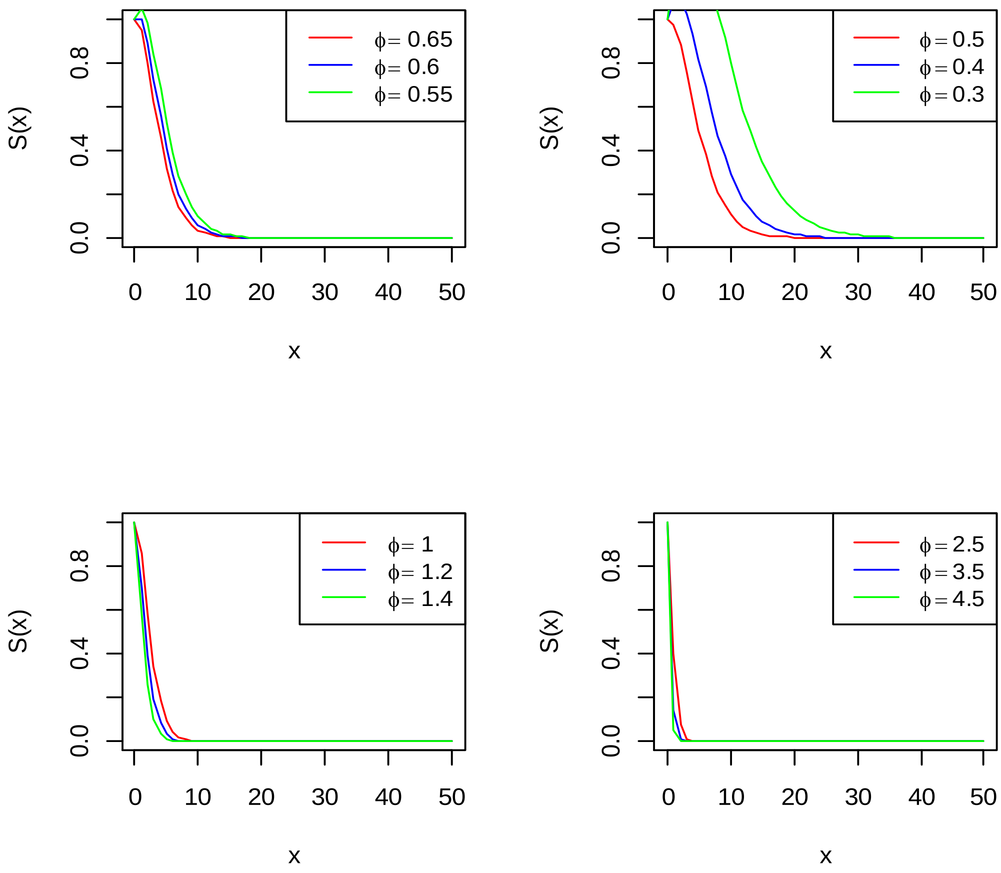

Fig. 2. Graphs of the survival function of the I-N Distribution for different values of ϕ

It is obvious from the graphs of $s(x)$ that the survival function is monotonically decreasing function and attend a constant rate at some higher values of x.

13 Applications and Goodness of Fit

In order to check the flexibility of the Iwok-Nwikpe distribution, real life data sets were used to fit the distribution. The goodness of fit was compared with that of Amarendra, Sujatha, exponential Lindley and Shanker distributions using the following information criteria: Bayesian Information Criterion (BIC), Akaike Information Criterion(AIC) and Akaike Information Criterion Corrected (AICC). The distribution with the least BIC, AIC and AICC is regarded as the most flexible distribution for any data set. The information criteria used are defined as follow:

$$AIC = -2\ln L + 2k\ , BIC = -2\ln L + k\ln n, AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

Where k is the number of parameters and n is the sample size.

14 Data Sources

The first data set used is the strength data of glass of the airplane window prearranged by Fuller et al. (1994) in Shanker (2015).

The second data set is data on tensile potency, measured in GPa, of some carbon fibres reported by Bader and Priest (1982) in Shanker (2016). The data set consist of 69 measurements

The third data is the relief times measured in minutes of twenty(20) patients receiving an pain reliever. The data set was first reported by Gross and Clark (1975) in Shanker (2016).

The maximum likelihood and goodness of fit criteria were computed using the R-Software.

First Data set:

18.83	20.80	21.66	23.03	23.23	24.05	24.32	25.50	25.52	25.80
26.69	26.77	26.78	27.05	27.67	29.90	31.11	33.20	33.73	33.76
33.89	34.76	35.75	35.91	36.98	37.08	37.09	39.58	44.05	45.29
45.38									

Second Data set:

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958
1.966	1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179
2.224	2.240	2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382
2.382	2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535	2.554
2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684	2.697	2.726
2.770	2.773	2.800	2.809	2.818	2.821	2.848	2.880	2.954	3.012
3.067	3.084	3.090	3.096	3.128	3.233	3.433	3.585	3.585	

Third Data set:

1.1 1.4 1.3 1.7 1.9 1.8 1.6 2.2 1.7 2.7 4.1 1.8 1.5 1.2 1.4 3 1.7 2.3 1.6 2

Table 1. Table of comparison for the five different distributions using three data sets

Data Set	Model	Parameter Estimate	$-2\ln L$	AIC	BIC	$AICC$
First	Exponential	0.0324	274.53	276.53	277.96	276.67
	Lindley	0.0629322	252.9932	255.9942	257.42331	256.1332
	Shanker	0.06471203	252.30	254.30	255.80	254.50
	Sujatha	0.09561026	241.5032	243.5031	244.9432	243.64301
	Amarendra	0.09706217	240.6818	242.6818	244.10	242.80
	Iwok-Nwi.	0.09588173	241.30	241.2062	242.51	241.20
Second	Exponential	0.407941	261.7432	263.7411	265.9655	263.80112
	Lindley	0.65900001	238.3667	240.3659	242.6134	240.44
	Shanker	0.6581	233.0054	235.0054	237.2376	235.01
	Sujatha	0.9361	221.6088	223.6088	225.8355	223.6688
	Amarendra	1.2443	207.947	209.947	209.7858	210.007
	Iwok-Nwi.	0.0767	198.4607	200.4608	201.3401	205.023
Third	Exponential	0.5263	65.700	67.700	68.7000	67.90
	Lindley	0.8161	60.500	62.500	63.4900	62.720
	Shanker	0.8039	59.77	61.783	61.0810	61.840
	Sujatha	1.15690	59.500	61.700	61.7200	61.720
	Amarendra	1.480700	55.635	57.6410	58.6300	57.860
	Iwok-Nwi.	1.3654110	49.678	51.7188	52.6300	51.0320

The result in Table 1 above clearly shows that the Iwok-Nwikpe distribution has the smallest BIC, AIC and AICC for the three data sets.

15 Conclusion

A new one parameter distribution named Iwok-Nwikpe distribution was derived in this paper. The new distribution is a two component mixture of $\text{gamma}(2, \phi)$ and $\text{gamma}(3, \phi)$ distributions. Some statistical properties of the new distribution have been derived. The graph of the p.d.f. shows that the Iwok-Nwikpe distribution could model heavily skewed data sets. The goodness of fit of the Iwok-Nwikpe distribution was determined using some real data sets. The results in Table 1 confirmed that the Iwok-Nwikpe distribution gave the best fit to the data sets used for this study.

Competing Interests

Authors have declared that no competing interests exist.

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