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New Modified Efficient Variance Estimators for the Estimation of Population Variance in Survey Sampling

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

For efficient and precise estimate of population variance, we have proposed some new ratio type variance estimators in the current research work by incorporating the linear combination of conventional and non-conventional parameters of auxiliary information. Bias and mean square error have been computed up to the first order of approximation. Numerical demonstration has been carried out to test the efficiency of new proposed estimators against existing estimators.

Keywords: Sample random sampling; bias; MSE; HL; MR; efficiency.

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1. INTRODUCTION

Population variance is one of the key indicators to provide the knowledge about the variations. As the variations occurs naturally almost in every field of science, even in our day-to-day life. To have the accuracy about the levels of these variations, we need precise and efficient estimators for the estimation of finite population variance. Especially when the estimates are being operated in terms of ratios, factors like the differences in ratios, skewness and presence of outliers cause biases in the estimation. As it is well known fact that accurate and precise estimates help us in decision making, cost savings as well as in policy making. Thus in order to have the reliable and precise estimates of population parameters, finding an efficient estimator is a common practice of authors to address the issue of precision. Using auxiliary data, Isaki (1983) introduced the ratio and regression estimator. Later, Kadilar& Cingi(2006a) proposed ratio type variance estimator by utilizing coefficient of skewnes as supplementary information to enhance the efficiency of estimator. Similarly, A Class of estimators was also proposed by Subramani J. and Kumarapandiyan (2012) who have modified estimators using quartiles and quartile deviation as supplementary information to improve the efficiency of estimators. Similarly Bhat M. A *et a*l., (2018) have used known values of auxiliary information. Khan & Shabir . J (2013) has used quartiles as auxiliary information to enhance the efficiency of estimators. Prased . B.and Singh H. P (1999) has introduced improved ratio type variance estimators. Yadav S.K. Sharama et al, have incorporated tri-mean and quartile as auxiliary information to minimize the bias.

Singh H.P Taylor *et al*., have used power transformation to enhance the efficiency of variance estimators. By employing the modification technique to achieve the goal of efficiency, we have developed some new efficient estimators for the estimation of population variance.

2. REVIEW OF ESTIMATORS IN LITERATURE

Let the finite population under survey be $U=\{U_1,U_2,...,U_N\}$, consists of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i=1,2,3........N$, giving a vector $Y=\{y_1,y_2,...,y_N\}$. The goal is to estimate the populations mean $\bar{Y}=\frac{1}{N}\sum_{I=1}^{N} y_i$ and its variance $S_Y^2=\frac{1}{N-1}\sum_{I=1}^{N}\left(y_i-\bar{y}\right)^2$ on the basis of random sample selected from a finite population ‘U’.

$N = Population\ size$,

$n = sample\ \ size$.

$\gamma = \frac{1}{n}$

Y = *Study* Variable,

X = Auxiliary Variable,

$\bar{X}, \bar{Y}$ = *Population means for auxiliary and study variable* ,

$\bar{x}$, $\bar{y}$ = *Sample means for auxiliary and study variable* ,

S_Y^2 , S_x^2 = *population Variances for study and auxiliary variable* ,

s_y^2 , s_x^2 = *Sample variances for study and auxiliary variable* ,

C_x , C_y = *Cofficient of variations,*

ρ = *Cofficient of Correlation between auxiliary and study variable* ,

$\beta_{2(x)}$ = *Cofficient of kurtosis for auxiliary variable* ,

$\beta_{2(y)}$ = *Cofficient of kurtosis for Study Variable*

$HL = Hodg's - Lehmanne, MR = Mid-range$ *for auxiliary variable*

$$\lambda_{rs}=\frac{\mu_{rs}}{\mu_{rs}^{\frac{r}{2}}\,\mu_{rs}^{\frac{s}{2}}},\ \mu_{rs}=\frac{1}{N-1}\sum_{i=1}^{N}\left(Y-\bar{Y}\right)^{r}\left(X-\bar{X}\right)^{s}$$

3. EXISTING ESTIMATORS FROM THE LITERATURE

Ratio type Variance estimator proposed by Isaki (Isaki, 1983):

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2}$$

Bias ($(\hat{S}_R^2) = \gamma S_y^2 \left[\left(\beta_{2(x)} - 1 \right) - (\lambda_{22} - 1) \right]$

MSE ($(\hat{S}_R^2) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1 \right) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1) \right]$

Ratio type Variance estimator proposed by Upadhayaand Singh (1999):

$$\hat{S}_{US}^2 = s_y^2 \left[\frac{S_x^2 + \beta_{2x}}{s_x^2 + \beta_{2x}} \right]$$

Bias ($(\hat{S}_{US}^2) = \gamma S_y^2 A_{US} \left[A_{US} \left(\beta_{2(x)} - 1 \right) - (\lambda_{22} - 1) \right]$

MSE ($(\hat{S}_{US}^2) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1 \right) + A_{US}^2 (\beta_{2(x)} - 1) - 2A_{US} (\lambda_{22} - 1) \right]$

Ratio type Variance estimator proposed by Kadilar and Cingi(2006a):

$$\hat{S}_{kc1}^2 = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x} \right]$$

Bias ($(\hat{S}_{kc1}^2) = \gamma S_y^2 A_1 \left[A_1 \left(\beta_{2(x)} - 1 \right) - (\lambda_{22} - 1) \right]$

MSE ($(\hat{S}_{kc1}^2) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1 \right) + A_1^2 (\beta_{2(x)} - 1) - 2A_1 (\lambda_{22} - 1) \right]$

Ratio type Variance estimator proposed by Subramani and Kumarapandiyan(2012):

$$\hat{S}_{kc1}^2 = s_y^2 \left[\frac{S_x^2 + Q_d}{s_x^2 + Q_d} \right]$$

Bias ($(\hat{S}_{kc1}^2) = \gamma S_y^2 A_1 \left[A_1 \left(\beta_{2(x)} - 1 \right) - (\lambda_{22} - 1) \right]$

MSE ($(\hat{S}_{kc1}^2) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1 \right) + A_1^2 (\beta_{2(x)} - 1) - 2A_1 (\lambda_{22} - 1) \right]$

4. PROPOSED ESTIMATORS: WE HAVE DERIVED HERE THE BIAS AND MEAN SQUARE ERROR OF THE PROPOSED ESTIMATOR $\hat{S}_{MSi}^2;\ i = 1, 2$ TO FIRST ORDER OF APPROXIMATION AS GIVEN BELOW:

1. $$\hat{S}_{MS_1}^2 = s_y^2 \left[\frac{S_x^2 + MR + \{\frac{S_x + C_x}{\beta_{1x}}\}}{s_x^2 + MR + \{\frac{S_x + C_x}{\beta_{1x}}\}} \right]$$

2. $$\hat{S}_{MS_1}^2 = s_y^2 \left[\frac{S_x^2 + MR + \{\frac{S_x + \beta_{2x}}{\beta_{1x}}\}}{s_x^2 + MR + \{\frac{S_x + \beta_{2x}}{\beta_{1x}}\}} \right]$$

3. $\hat{S}^2_{MS_1} = s^2_y \left[\frac{S^2_x + MR + \{\frac{S_x + HL}{\beta_{1x}}\}}{s^2_x + MR + \{\frac{S_x + HL}{\beta_{1x}}\}} \right]$

Let $e_0 = \frac{s^2_y - S^2_y}{S^2_y}$ and $e_1 = \frac{s^2_x - S^2_x}{S^2_x}$. Further we can write $s^2_y = S^2_y(1+e_0)$ and $s^2_x = S^2_x(1+e_1)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0\ ,\quad E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1)\ ,\quad E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1)\ ,\ E[e_0 e_1] = \frac{1-f}{n}(\lambda_{21} - 1)$$

The proposed estimator $\hat{S}^2_{MSi}\ ;\ i = 1,2,3$ is given below:

$$\hat{S}^2_{MSi} = s^2_y \left[\frac{S^2_x + \alpha a_i}{s^2_x + \alpha a_i} \right]$$

$$\Rightarrow\ \hat{S}^2_{MSi} = s^2_y(1+e_0)\left[\frac{S^2_x + \alpha a_i}{s^2_x + e_1 S^2_x + \alpha a_i} \right] \Rightarrow\ \hat{S}^2_{MSi} = \frac{S^2_y(1+e_0)}{(1+A_{MSi}e_1)} \text{ Where } A_{MSi} = \frac{S^2_x}{S^2_x + \alpha a_i}\ ,$$

$$a_i = [(\frac{S_x + C_x}{\beta_{1x}})], [(\frac{S_x + \beta_{2x}}{\beta_{1x}})], [(\frac{S_x + HL}{\beta_{1x}})];\ i = 1,23$$

$$\Rightarrow\ \hat{S}^2_{MSi} = S^2_y(1+e_0)(1+A_{MSi}e_1)^{-1}$$

$$\Rightarrow\ \hat{S}^2_{MSi} = S^2_y(1+e_0)(1 - A_{MSi}e_1 + A^2_{MSi}e_1^2 - A^3_{MSi}e_1^3 + ...) \tag{4.1}$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}^2_{MSi} = S^2_y + S^2_y e_0 - S^2_y A_{MSi} e_1 - S^2_y A_{MSi} e_0 e_1 + S^2_y A^2_{MSi} e_1^2 \tag{4.2}$$

$$\hat{S}^2_{MSi} - S^2_y = S^2_y e_0 - S^2_y A_{MSi} e_1 - S^2_y A_{MSi} e_0 e_1 + S^2_y A^2_{MSi} e_1^2 \tag{4.3}$$

By taking expectation on both sides of (4.5), we get

$$E(\hat{S}^2_{MSi} - S^2_y) = S^2_y E(e_0) - S^2_y A_{MSi} E(e_1) - S^2_y A_{MSi} E(e_0 e_1) + S^2_y A^2_{MSi} E(e_1^2) \tag{4.4}$$

$$Bias(\hat{S}^2_{MSi}) = S^2_y A^2_{MSi} E(e_1^2) - S^2_y A_{MSi} E(e_0 e_1) \tag{4.5}$$

$$Bias(\hat{S}^2_{MSi}) = \gamma\, S^2_y A_{MSi} [A_{MSi}(\beta_{2(x)} - 1) - (\lambda_{21} - 1)] \tag{4.6}$$

Squaring both sides of (4.6) and neglecting the terms more than 2nd order and taking expectation, we get

$$E(\hat{S}^2_{MSi} - S^2_y)^2 = S^4_y E(e_0^2) + S^4_y A^2_{MSi} E(e_1^2) - 2 S^4_y A_{MSi} E(e_0 e_1) \tag{4.7}$$

$$MSE(\hat{S}^2_{MSi}) = \gamma\, S^4_y [(\beta_{2(y)} - 1) + A^2_{MSi}(\beta_{2(x)} - 1) - 2 A_{MSi}(\lambda_{21} - 1)] \tag{4.8}$$

5. EFFICIENCY CONDITIONS

Here, we have derived the efficiency conditions of proposed estimators with other existing estimators under which proposed estimators have performed better than the existing estimators.

The bias and mean square error of existing ratio type estimators up to the first order of approximation is given by

$$Bias\left(\hat{S}_K^2\right)=\gamma S_y^2\varphi_K\left[\varphi_K\left(\beta_{2x}-1\right)-\left(\lambda_{21}-1\right)\right] \tag{5.1}$$

$$MSE\left(\hat{S}_K^2\right)=\gamma S_y^4\left[\left(\beta_{2y}-1\right)+\varphi_K^2\left(\beta_{2x}-1\right)-2\varphi_K\left(\lambda_{21}-1\right)\right] \tag{5.2}$$

$\varphi_K = Existing.cons\tan t$

$, K=1,2,3,4........$

Bias, MSE and constant of proposed estimators is given by

$$Bias\left(\hat{S}_P^2\right)=\gamma S_y^2\varphi_P\left[\varphi_P\left(\beta_{2x}-1\right)-\left(\lambda_{21}-1\right)\right] \tag{5.3}$$

$$MSE\left(\hat{S}_P^2\right)=\gamma S_y^4\left[\left(\beta_{2y}-1\right)+\varphi_P^2\left(\beta_{2x}-1\right)-2\varphi_P\left(\lambda_{21}-1\right)\right] \tag{5.4}$$

$\varphi_P = proposed.cons\tan t$

$P=1,2,3............$

From Equation (5.2) and (5.4), we have

$$MSE\left(\hat{S}_P^2\right)\le MSE\left(\hat{S}_K^2\right) if\ \lambda_{21}\ge 1+\frac{\left(\varphi_P+\varphi_K\right)\left(\beta_{2x}-1\right)}{2}$$

$$MSE\left(\hat{S}_P^2\right)\le MSE\left(\hat{S}_K^2\right)$$

$$\gamma S_y^4\left[\left(\beta_{2y}-1\right)+\varphi_P^2\left(\beta_{2x}-1\right)-2\varphi_P\left(\lambda_{21}-1\right)\right] \le \gamma S_y^4\left[\left(\beta_{2y}-1\right)+\varphi_K^2\left(\beta_{2x}-1\right)-2\varphi_K\left(\lambda_{21}-1\right)\right]$$

$$\Rightarrow \left[\left(\beta_{2y}-1\right)+\varphi_P^2\left(\beta_{2x}-1\right)-2\varphi_P\left(\lambda_{21}-1\right)\right] \le \left[\left(\beta_{2y}-1\right)+\varphi_K^2\left(\beta_{2x}-1\right)-2\varphi_K\left(\lambda_{21}-1\right)\right]$$

$$\Rightarrow \left[\varphi_P^2\left(\beta_{2x}-1\right)-2\varphi_P\left(\lambda_{21}-1\right)\right] \le \left[\varphi_K^2\left(\beta_{2x}-1\right)-2\varphi_K\left(\lambda_{21}-1\right)\right]$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(\varphi_P^2-\varphi_K^2\right)\left[-2\varphi_P\left(\lambda_{21}-1\right)\right] \le \left[-2\varphi_K\left(\lambda_{21}-1\right)\right]$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2-R_K^2\right)\left[-2\left(\lambda_{21}-1\right)\left(\varphi_P-\varphi_K\right)\right] \le 0$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(\varphi_P^2-\varphi_K^2\right) \quad \le \left[2\left(\lambda_{21}-1\right)\left(\varphi_P-\varphi_K\right)\right]$$

$$\Rightarrow \left(\beta_{2x}-1\right) \le \frac{2\left(\lambda_{21}-1\right)\left(\varphi_P-\varphi_K\right)}{\left(\varphi_P^2-\varphi_K^2\right)}$$

$$\Rightarrow \left(\beta_{2x}-1\right) \le \frac{2\left(\lambda_{21}-1\right)\left(\varphi_P-\varphi_K\right)}{\left(\varphi_P-\varphi_K\right)\left(\varphi_P+\varphi_K\right)}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(\varphi_P+\varphi_K\right) \le 2\left(\lambda_{21}-1\right) \tag{5.5}$$

By solving equation (5.5), we get

$$MSE\left(\hat{S}_P^2\right)\le MSE\left(\hat{S}_K^2\right)\ if \quad \lambda_{21}\ge 1+\frac{\left(\varphi_P+\varphi_K\right)\left(\beta_{2x}-1\right)}{2}$$

6. NUMERICAL ILLUSTRATION

We use the data set of Bhat et al., (2018, page228) in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories is denoted by Y (study variable). The Population parameters are given below

$N = 80,\ n = 20,\ \rho = 0.9413,\ \bar{X} = 11.2646,\ \bar{Y} = 51.8264,\ S_y = 18.3549,\ S_x = 8.4563.\ C_x = 0.7507$

$C_y = 0.3542, \beta_{1x} = 1.05\ \ \beta_{2x} = 2.8664,\ \beta_{2y} = 2.2667, Q_1 = 5.1500\ , Q_2 = 7.5750,\ Q_3 = 16.975,\ Q_d = 5.9125,$

$Q_r = 11.825,\ Q_a = 11.0625\ , HL = 10.405, MR = 17{,}955,\ \lambda_{22} = 2.2209$

Table 1. Bias and mean square error of existing and proposed estimators

Estimators	**Population-1**		**PRE%**
Existing Estimators	Bias	MSE	PRE%
Isaki (1983)	14.92	7705.22	285.30
Upadhyaya and Singh (1999)	13.50	6327.13	290.97
Kadilar and Cingi (2006)	14.79	7665.78	293.47
Subramani and Kumarapandiyan	12.41	6207.29	234.27
Proposed Estimators	Bias	MSE	238,93
Proposed Estimator-1	1.64248	2700.69	240.98
Proposed Estimator-2	1.27416	2648.04	283.84
Proposed Estimator-3	0.18925	2625.53	289.48

7. CONCLUSION

The new proposed efficient estimators modified by incorporating the linear combination of conventional and non-conventional parameters as auxiliary information have performed better than the existing estimators. The improvement can be easily accessed from the Table 1, by comparing the bias and MSE and PRE of existing and new modified estimators.

Hence new modified estimators may be preferred over existing estimators for use in practical applications.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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