Journal of Education, Society and Behavioural Science

33(11): 93-111, 2020; Article no.JESBS.64155
ISSN: 2456-981X
(Past name: British Journal of Education, Society & Behavioural Science, Past ISSN: 2278-0998)

Learning and Engagement in the Flipped Classroom of Analytical Mathematics

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/JESBS/2020/v33i1130275
Editor(s):
(1) Prof Redhwan A. Al-Naggar, UniversitiTeknologi MARA, Malaysia.
(2) Chih-Wei Pai, Taipei Medical University, Taiwan.
Reviewers:
(1) Aline Beckmann Menezes, Universidade Federal do Pará, Brazil.
(2) Jurandyr Santos Nogueira, Federal University of Bahia, Brazil.
Complete Peer review History: http://www.sdiarticle4.com/review-history/64155

Original Research Article

Received 20 October 2020
Accepted 23 December 2020
Published 30 December 2020

ABSTRACT

Learning techniques have changed over time in order to try and improve student engagement across different subjects in higher education. Mathematics has dominantly adhered to certain learning methods that use a more conventional approach. Interactive and active learning in Mathematics tend to be more common in further education yet, university level Mathematics is more complex, heavy in content and poses more difficulty in applying active learning approaches as a passive approach of traditional lectures has always been applied. The issues of learning problems in mathematics is ignored and the lack of metacognitive awareness of mathematical thinking and problem-solving skills seem to persist despite differences amongst educators on an effective learning methodology.

Following the reform movement in mathematics education in the mid 1980's, resulting from the dissatisfaction of conventional approaches, recommending the restructuring of mathematical delivery marked the need for modifications in teaching methodology. Employing multiple models to deliver lessons may implement the changes needed to drive student engagement and satisfaction to improve the experience in learning mathematics. In order for these methods to become applicable and effective in students' experiences in mathematical education, educators need to be encouraged to present active learning techniques so that students can begin to facilitate their own learning which can be done through introducing approaches specific to the individual such as student-centred approaches.

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This paper evaluates the techniques used by mathematicians to deliver lessons and how it reflects on learning and engagement of students in comparison to the flipped classroom approach which inverts the common traditional lecture style used in classrooms. The flipped classroom model in this study is adopted to a topic from the university foundation level module, Analytical Mathematics, whereby results from the quantitative analysis undertaken show a decrease in the success of students' performance suggesting a lesser impact on improved learning. With regards to engagement, observations from the qualitative analysis of the study highlight positive aspects of the flipped classroom model, specifically an optimistic engagement amongst peers.

Keywords: Flipped classroom; mathematics; learning and engagement.

1. INTRODUCTION

This paper is divided into three sections. The first part introduces the different educational models endorsed by a literature review analysing the research of existing sources on the flipped classroom. Section two focuses on the methodology of the adopted flipped class intervention, the results obtained, and the drawbacks of the flipped classroom approach followed by a discussion. The final section comprises of conclusions based on the intervention as well as relating the findings to current literature articles. The strengths and limitations of the flipped classroom approach, contributions and future work are also explored within this latter section.

This paper investigates how introducing learning models such as 'the flipped classroom' creates 'improved engagement and learning' within classrooms focusing on communication between students and techniques used to improve experiences for students and encouraging them to create their own learning opportunities. The study focus is on aspects of engagement deferring from performance of students and highlighting satisfaction and experiences achieved by adapting a new learning model.

The interventions aims to answer the following research questions: "Does the flipped classroom model work for an introductory university level mathematics module?", "How effective is the flipped classroom model in improving its student satisfaction?" and "Do students in this sector of education perform better as a result of the flipped classroom?". These questions help answer the main question of this investigation and further explore the components of "improved learning and engagement" supplementing whether or not the flipped classroom is able to achieve this.

Education has been a foundation of society throughout generations. The fundamentals of learning have often remained the same, whether it be in education or any other branches of society. Educators have the responsibility of delivering this knowledge and ensuring that students perform well. Classrooms have often remained teacher-centred and focused on students' success rather than needs and preferences. Within the sector of higher education in mathematics, there persists a lack of engagement in the classroom which perhaps needs to improve so as to better the learner performance and achievements within the sector.

There are many different approaches that are used in learning; however, the foundations of these approaches are behaviourist, cognitive, humanistic and constructivist [1]. Understanding the depths of these theories, one can conclude that learning can be either seen as passive or active. Historically, approaches used to deliver mathematics has been passive and based on instruction from an educator obliging by the teacher-centred approach that the behaviourist theory identifies.

However, in modern education tactics, the theories that abide are more cognitive and humanistic approaches, these that are implemented in classrooms. With these approaches there is a deeper understanding on how students learn and engage with content taking into consideration the thinking and mental capabilities of students by their educators whilst delivering lessons. These approaches can be seen as more active as they require the individual to take charge with their own learning. Teaching methods such as collaborative learning in particular collaborative inquiry are the types of active learning that improve student engagement in mathematics. Students are viewed as active agents inquiring knowledge individually [2]. Students inquiring and seeking knowledge to further their understanding restores the idea that engagement and students' individual needs lays the groundwork for success. The further steps in

which a student takes to convert the knowledge to become applicable learning extends further than inquiring knowledge and contributes to their success [3].

The idea of the flipped classroom takes on this approach as it relies on student further expanding their knowledge by questioning outside the classroom in order to attack problems in the classroom.

1.1 Defining the Flipped Classroom

The traditional classroom is often used to deliver lessons in mathematics. However, in modern society and in diverse disciplines, different techniques are used in teaching such as the flipped classroom approach to deliver lessons. The flipped learning is a reversed notion of the traditional classroom approach where students use class time to learn and be tested and home time to further expand their knowledge [4].

The Higher Education Academy describes the flipped classroom approach as a pedagogical approach in which the traditional notion of classroom-based learning is inverted. This means that students are expected to work independently by learning material prior to lessons, where lesson time is then used to further expand the knowledge that they have accrued through active learning by discussions with peers and problem-solving activities.

The use of the flipped learning approach in mathematics means that engagement in the classroom amongst peers is improved deviating away from the traditional note taking typically used by lecturers. Educators can flip the classroom, nevertheless, in order for it to become flipped it must have the pillars of flexible environment, learning culture, intentional content and the professional educator which is more demanding in a flipped classroom as it relies on teachers' observations and feedback of the students [5]. The simpler definition of the flipped classroom is that it is an inverted model of the traditional classroom. However, the proposed definitions employ interactive techniques inside the classroom [4].

Fig. 1, demonstrates the literal meaning of the flipped classroom [4], whereas Fig. 2 provides a broader definition where the structure teachers use in the classroom to deliver lessons are dependent on the instructor's educational philosophy, classroom demographic and the schools' aim [6]. The scale interprets educators as the main authority figure in the teacher-centred model while students are seen as passive and viewed as "empty vessels" with the purpose of absorbing information fed to them by their instructor using traditional methods. "Teach" describes such methods as having less interaction amongst students and the success is also measured using traditional progress tracking such as objectively scored tests and assessments [6].

In contrast, the student-centred approach, sees the educationalist remaining as the authoritative figure in the classroom whilst students pay an equally active role in their learning. The educators' primary role changes and focuses on facilitating the students learning and comprehension of the content. On the scale used by "teach", the use of technology plays a role in where teaching techniques lie [6].

The flipped classroom approach is considered to be a high-tech type of teaching method. While flipped learning can be measured to be student centred, "teach" places flipped classroom in the lower extremities of the teacher-centred scale. Flipped classroom is described as a structure that uses pre-recorded lessons at home and completing work in class, where the traditional notion of lectures in class and homework at home is switched. The content in which students use at home can be created by the instructor or an online source. This approach allows students to work at their own pace, however, due to instructions given by teachers and content being based on their ideas, the flipped classroom approach lacks potential in being student-centred.

Fig. 2 demonstrates how far the flipped classroom deviates from the traditional lecture (Direct instruction). On this scale 'Direct Instruction' appears on the extreme ends of both low tech and teacher centred. This shows the lack of interaction between students and how they act as empty vessels.

In contrast, the 'Flipped Classroom' appears on the high-tech side as it requires students to inquire using video tutorials and further research if necessary. Flipped classroom is more on the teacher centred side of the scale. Although, it produces efforts to being student-centred, it still functions on the idea of the educator. Content students watch is often recorded by teachers and materials chosen are from the teacher and still depicted by content they believe students should learn [6].

Table 1: Restricted definition of the flipped classroom.

Style	Inside Class	Outside Class
Traditional	Lectures	Practice Exercises & Problem Solving
Flipped	Practice Exercises & Problem Solving	Video Lectures

Table 2: Broader definition of the de-facto flipped classroom.

Inside Class	Outside Class
Questions & Answers	Video Lectures
Group-Based/Open-Ended Problem Solving	Closed-Ended Quizzes & Practice Exercises

Fig. 1. Defining the flipped classroom (adapted from [4]).

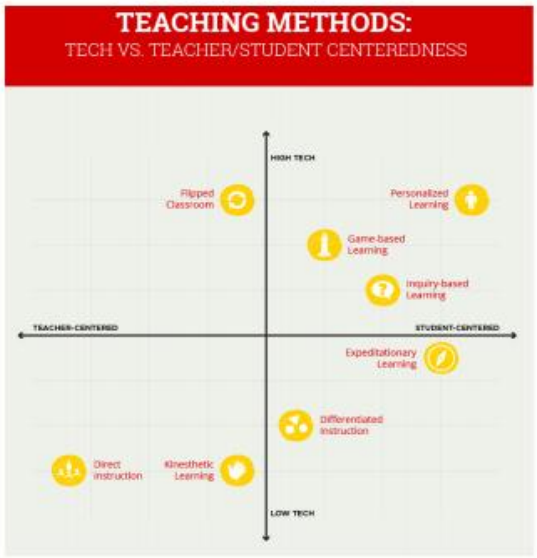

Fig. 2. Teacher-student centredness VS tech (adapted from [6])

Throughout this paper, the objective is to understand the effectiveness of introducing the flipped classroom model in a mathematics higher education classroom. In modern society, with various new techniques being introduced, it can be considered as beneficial to implement such approaches to different subjects and to investigate whether introducing such techniques can be feasible for the subject of mathematics. On this purpose, the aim of this research paper is to investigate how effective active techniques such as the flipped classroom is, in a university introductory level mathematics module.

2. LITERATURE REVIEW

The use of active learning in higher education has obtained positive results. In a study testing how active learning impacted student performance, results showed that active learning increased by half a Standard Deviation, while lectures increased failure rates by 55 percent [7]. An attempt to implement active learning is through the flipped classroom approach. The strengths of the flipped classroom stem from the strength of the overall success of active learning approaches as it encourages educators to implement such techniques into their curriculum. Introducing these interactive learning techniques may be easier in subjects that are more discussion based and thought oriented. Subjects such as mathematics have a more direct approach where the methodology and answers are clear and so the use of more complex teaching techniques typically are not desired by educators. Still, teachers assume there is a 'one size fits all' approach and therefore abide to traditional teaching styles.

Academics often deliver lessons without understanding how students learn best [1]. Educators need to implement techniques which

work for the individual. The flipped classroom approach is a type of learning that focuses on the individual by allowing students to find opportunities to engage in class by allowing discussions to take place amongst peers [8]. Many learning difficulties experienced by students in undergraduate courses is related to passive style used in lectures, this can be solved by implementing active learning techniques [9].

The flipped classroom model addresses one of the biggest challenges of learning mathematics a result of the traditional lecture model, solving practice problems. Solving problems at home where students have no support from peers or an instructor [7]. Research on a college algebra class showed that the flexibility and independence of the flipped classroom meant that students could revisit videos to address anything they didn't understand for as long as they needed [10]. The flipped class model resulted in significantly higher results in observations of university students in California, 669 students with results showing a passing rate of 66 percent for the flipped group while only 57 percent passed from the traditional group [11].

Similarly, research on a Calculus I and Calculus II module showed that students who were self-efficacious performed better both independently and in class. However, students in the flipped classroom were not homogenous, students' experiences were independent to their efforts and abilities [12].

A study on undergraduate Calculus students which had a flipped and lecture group showed a higher normalized gain in the flipped classroom group, significantly higher outcomes on the engagement survey and better performance on the final exam of the course [13]. In a study at an Australian University, students felt that in terms of engagement, flipped learning would lead to a greater understanding of the material [14]. The teaching techniques used in the classroom are dependent on the educator and the methods they choose to employ in their lectures. While the traditional lecture style is often used, faculty do recognise the benefits of implementing different teaching techniques.

In a survey conducted on teaching-oriented faculty attending an education conference, results exhibited that while only 36 percent of respondents think that traditional lectures were a good teaching approach, a majority of 60 percent still chose to use them when delivering lessons [15]. This shows that although many educators think using alternative approaches to learning would be beneficial, most still employ traditional lectures. Methods used to teach faculty in the past, is what educators adopt as their own approaches assuming what worked for them works, for their students [16]. Student involvement in the classroom is used to design effective learning environments. Student involvement is defined as the amount of physical and psychological energy a student focuses towards their academic and social experience aside from the educators' efforts [17].

In order to see if the flipped classroom type of learning can be effective, it is necessary to measure the level of student involvement as it is a more active and interactive way of learning. While the traditional lecture style is less active and dependent on students absorbing the information given, the flipped classroom style requires students to be independent and proactive with their learning [17]. Introducing this in classrooms, means students have to adapt away from methods they are familiar with and rely on their own efforts and resources to achieve the outcome of learning they usually received in the classroom alone. These sources show that while many believe that the flipped classroom would be beneficial for students, very little research into the effectiveness exists and where results have been obtained, a combination of different models is found to be effective. Current research shows that the flipped classroom is more engaging then the traditional classroom, but some literature disagrees in introducing the flipped classroom in mathematics. This could be because while research exists, there is still a significantly lower amount of research done on the flipped classroom in higher education. The existing research typically focuses on mathematics at a lower level of education, in different subject areas or in different regions of the world which follow a different learning structure to the UK. A majority of this, focuses on the performance of students when such a model is applied, whereas a deduction on the impact of engagement is limited.

In view of this, by enhancing current literature, this paper allows originality and a provision of further information on how using active learning models such as the flipped classroom approach may improve engagement and learning in a mathematics classroom.

3. METHODOLOGY

While other research shows the effectiveness of the flipped classroom and the impact that active learning can have on students' attitudes, there is a limited amount of research on the engagement of the flipped classroom for the subject of mathematics. This study aims to reach to a deduction on how effective implementing active learning techniques can be.

The intervention of this paper focused on foundation year university students undertaking an Analytical Mathematics module. The group consisted of students taking degrees in Electronics and Electrical Engineering, Applied Sound Engineering and Mathematics and Statistics.

The sessions were conducted over three weeks, whilst studying the topic of partial fractions with an attendance of 17 students. Prior to this session, none of the students had used the flipped classroom model in mathematics before, although it was common practice in other modules. The aim of the study was to test how engaging students would find implementing new active learning techniques, specifically the flipped classroom model in the aforementioned mathematics module.

Students were assigned videos, notes and questions to practice at home on the topic of 'partial fractions' with examples on how to solve these questions. Further resources such as a website was provided for students who preferred notes over videos and the freedom of further exploring the topics in depth in order to gain understanding.

During the investigation, students were observed in the classroom to see their interactions and how they responded to the new learning methods introduced. These involved students discussing how they had found the methods introduced as well as tracking their progress and how they were finding the new model. It was also imperative that students were provided the correct resources to create a smooth running of the experiment. However, it was also essential that students were independent and accessed resources in their own time in order for the experiment to be successful. A wide variety of resources was available to students to match their intellect and preferred learning techniques which studying at home allowed them the freedom to achieve.

All students who participated in the study were given a survey with a series of questions asking them to evaluate the flipped classroom approach and the impact it has had on their learning.

Students were invited to participate in the study, which was voluntary due to ethical considerations and involved completion of the questionnaire with hand-written observations made by the educator. Questions ranged to include students' opinions on the content of their satisfaction about the model itself. This helped address the research question of how engaging students found the method to be.

In order to see how the flipped classroom model improved learning, the assessment marks of current student performance (Group A with flipped classroom) was compared to last year's cohort (Group B with no flipped classroom). Both groups had the same sample size with students having similar backgrounds in education as well as the same instructor. Group B (No flipped classroom) had also been taught the chapter of partial fractions. However, their lectures followed a traditional structure throughout the semester and all content was delivered in the classroom. One question of the assignment was on the topic of partial fractions with the rest of the questions adhering to a range of other topics that had also been covered in lectures. Similarly, Group A (Flipped classroom) also received a similar assignment with partial fractions questions and the rest of the questions covering a range of other topics within the syllabus.

In order to see how the newly introduced model improved their learning, the performance of Group B's (No flipped classroom) score on the partial fraction question relative to performance on the rest of the assignment questions was analysed. This was then compared to Group A's (Flipped classroom) performance on a similar question on the same topic of partial fractions relative to performance on the rest of the assignment. Hence, any differences in the level of difficulty of the partial fractions relative to other questions was taken into account during analysis.

In this context, the performance on the partial fraction question relative to the rest of the assignment questions was used as a tool to measure the effectiveness of the flipped classroom model, checking whether student

performance with the inclusion of the intervention improved the learning experience. Sofroniou and Poutos, referred to this metric as the performance ratio and it can be employed within this context to represent the effectiveness of the flipped classroom method [18]. The authors suggested this ratio to be calculated by examining the performance on the partial fraction question over the remaining questions from the assignment respectively.

$$\text{Performance Ratio} = \frac{\text{Total percentage marks on Partial Fraction Question}}{\text{Total percentage marks on Rest of Questions in the assessment}}$$

For example, using the results of Group B, a randomly selected student managed to score 6 out of 15 on the partial fraction question, 40% (6/15 x 100) was the number of marks allocated to the partial fraction question. 67 out of 85 marks were scored on the rest of the questions, therefore 78.82% ((67/85) x 100) was the number of marks allocated to the rest of the questions. Using this data, the performance ratio is thus 0.51 (40%/78.2%).

The performance ratio helps determine how well a student performed in the partial fractions section of their assignment. A performance ratio with a higher value than 1.0 suggests that the student performed better in the partial fraction section due to the value of the denominator being higher than the numerator.

This metric was used to decide how effective the flipped classroom approach is in the Analytical Mathematics module. The findings from the results of the performance ratio of the assignment, allowed to test whether the flipped classroom model improved learning, whilst on the other hand, the survey was able to assist in drawing deductions as to how the model improved engagement in the introductory mathematics module. Students participation in the study was voluntary due to ethical consideration as research involved observations of their activities, a survey, tests and analysis of their formal assignments.

4. RESULTS AND DISCUSSION

In order to investigate the effectiveness of the flipped classroom model, different representations of data were used. The investigation implemented both quantitative and qualitative methods to collect data.

4.1 Results for Qualitative Analysis: Survey

The survey focused on the students' perceptions on the flipped classroom model. It aimed to tackle one of the research questions of the paper, that is to understand whether the flipped classroom approach improves engagement, knowledge that current research literature lacks.

While students' general performance in class is important, it is also important to understand if the model improved overall satisfaction in the classroom. The questionnaire was the main source of feedback providing the qualitative prospect of this project as well as further quantitative research. The results of some of the questions from the survey, have been summarised below.

Fig. 3 depicts using bar charts the first four question responses to the survey. The first plot representing Question 1, shows that most of the students disagree with the statement "the flipped classroom is more engaging than the traditional lecture". Students who took part in the experiment felt that they preferred traditional lectures over the new introduced model of the flipped classroom.

Although students did not find the flipped classroom model to be more engaging then the traditional classroom, Question 2 depiction shows that students found that flipped classroom allowed them opportunities to engage with other students. The majority of students supported this statement with 35 percent agreeing and 24 percent strongly agreeing. This shows that in the case of this study, the flipped classroom model had achieved its purpose of increasing engagement between students with only 6 percent of students having selected "strongly disagree" when it came to engagement between students.

The first question from the survey shows that the overall engagement of the flipped classroom model, is not as engaging when compared to the traditional classroom. However, when focusing on the individual experience of the flipped classroom model, the second question's diagram shows that students find the model itself to be engaging when it comes to individual communication and experiences in the classroom between peers. Thus, the flipped classroom is able to create opportunities of engagement between students, a quality that is therefore absent in the traditional lecture and its standards of teaching.

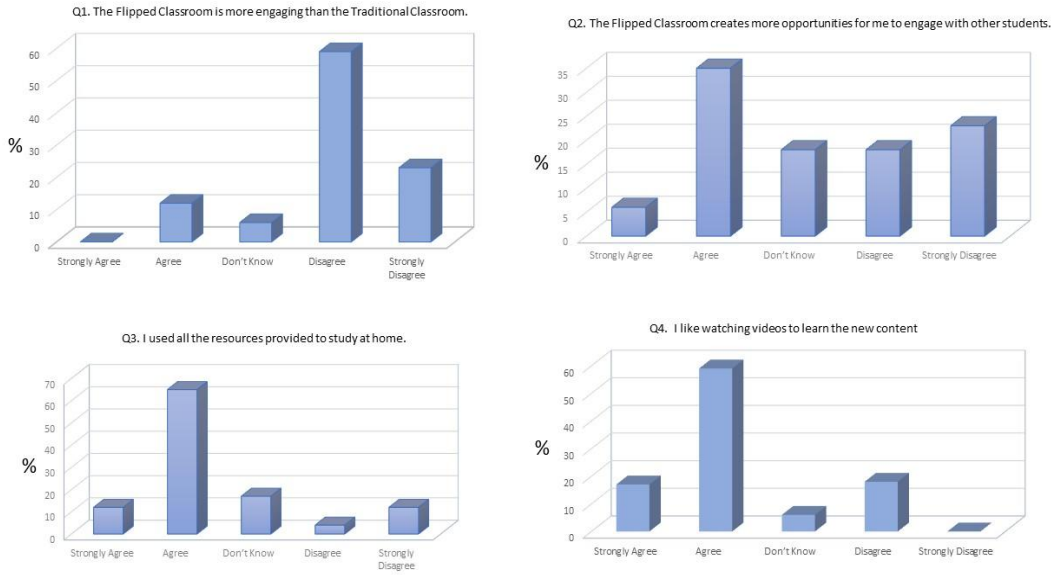

Fig. 3. Diagrammatic depiction of Question 1-4 from the survey

The investigation was dependent on students spending time at home studying independently with the resources recommended. From the class a strong 65 percent had chosen "agree" to the statement "I used all the resources provided to study at home" compared to only 6 percent having chosen "disagree". When compared to performance in the test results, this shows that the flipped classroom model had not had an impact on their performance and studying at home was not as beneficial.

The approach used to test the flipped classroom during the investigation was through students using videos to learn the topic of 'partial fractions'. In Fig. 3, specifically Question 3's bar chart, a high percentage of 59 agreed with the statement "I liked watching videos to learn the new content" as far as 17 percent even strongly agreeing. However, no students strongly disagreed with the statement showing that students found the new learning approach to be valuable.

Nevertheless, when compared to the traditional lecture, most students still felt that they preferred traditional teaching methods when it came to learning new topics in mathematics as represented in Fig. 3. Only 18 percent had disagreed with the statement "I would rather watch a traditional lecture than a lesson video" compared to 53 percent who preferred the traditional lecture over watching videos.

Whilst a majority of students felt that they appreciated watching videos, the figures showed students had a preference to the traditional lecture when it came to learning new content in mathematics.

Questions 5-8 from the survey are represented graphically in Fig. 4 and show that a high percentage of students felt that answering questions during lesson time was useful. This shows that while students may not advocate for flipped learning, aspects of the model were positive and could be incorporated in future lessons. While videos might not be the most useful tool to educate students outside the lessons, answering questions in the classroom is an aspect that the students support and find beneficial. Moreover, 56 percent of students said that they would recommend the flipped classroom to a peer. Less students (40% of the class) disagreed with the statement leaving 4% of the class size to remain unsure whether they would recommend the flipped classroom providing an indefinite answer.

While the traditional classroom remains as the preferred learning method, Question 8 validates the fact that students felt the experience of flipped classroom to be beneficial and would use the technique in future. Almost half of the

students agreed or strongly agreed with the statement "learning the flipped classroom will benefit me in my future education". The use of the model allowed them to be introduced to new ways of learning. Questions 6, 7 and 8 show that students' overall experience with the flipped classroom model was advantageous and aspects of the model had a positive impact on their learning.

Results from Questions 9-12 of the survey are summarised in Fig. 5 showing that students lacked motivation in learning using the flipped classroom. Only 31 percent of students agreed with the statement "I am more motivated to learn mathematics in the flipped classroom" with the majority finding the model to be less motivational.

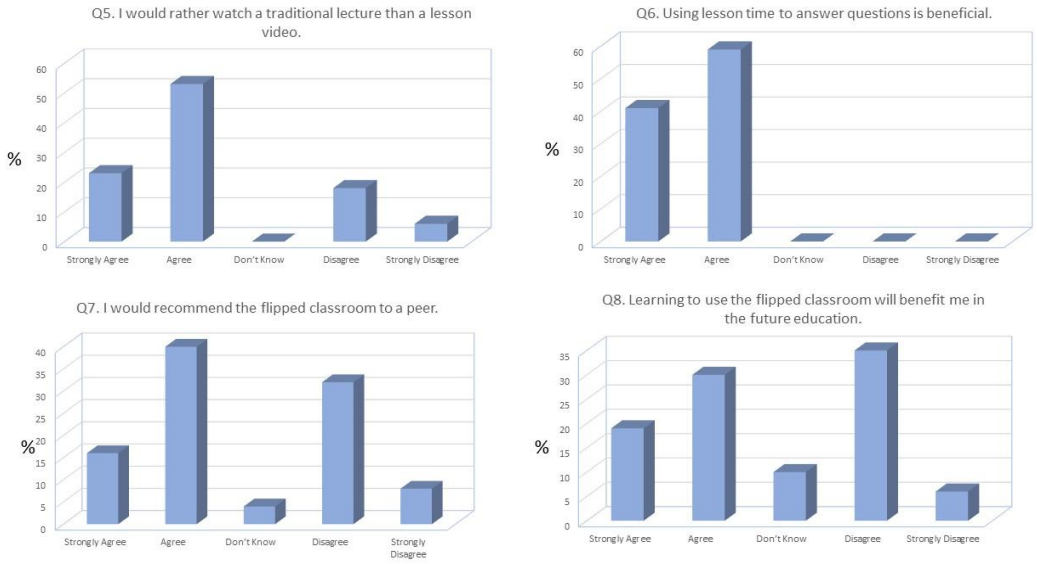

Fig. 4. Diagrammatic depiction of Question 5-8 from the survey

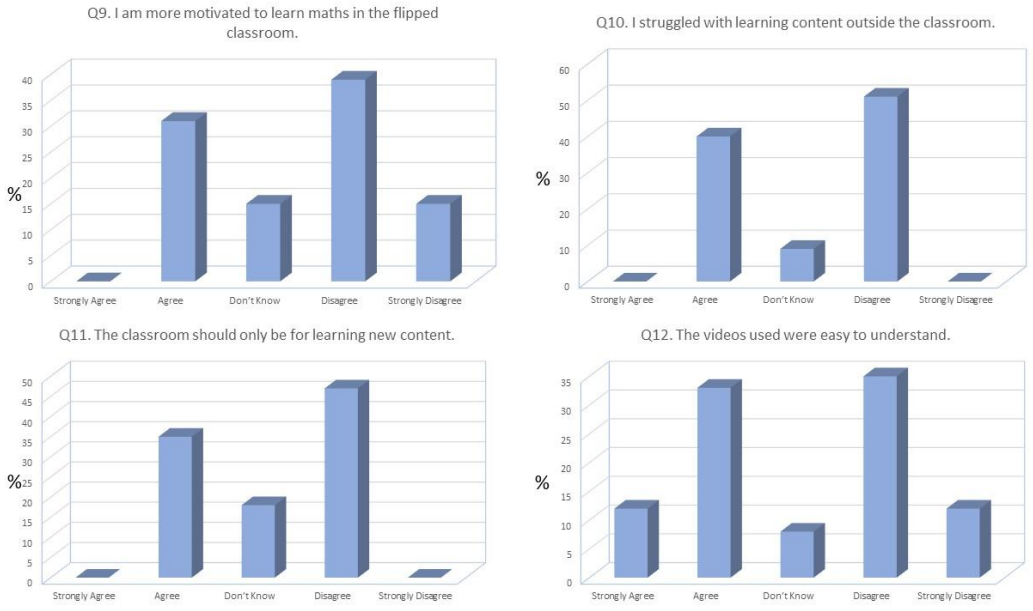

Fig. 5. Diagrammatic depiction of question 9-12 from the survey

From Fig. 5, it can be seen that almost half of the students disagreed that the lack of motivation was a result of struggles with learning content outside the classroom but yet 40 percent of the students confessed they struggled with the concept. Implying perhaps, that the flipped classroom approach can allow for difficulties and challenges in learning mathematical material.

35 percent of students who participated in the investigation agreed that the classroom should only be for learning new content and limited to a traditional structure. 47 percent disagreed and were open to flexibility, while the rest selected "Don't Know".

Question 12 represented also in Fig. 5 portrays that most students felt that the videos assigned to watch at home were not easy to understand. Almost half of the students did not understand the videos used to explain partial fractions.

4.2 Results for Quantitative Analysis: Performance ratio

The survey was used to interpret students' opinion on the newly introduced model and the impact it had on their learning experience. To further conclude how effective the model was in students' learning, the assignment results for Group A and Group B were gathered in order to produce a valuable conclusion for the study.

To determine whether the flipped classroom model was effective in achieving the objective of improving learning in mathematics, it was important to investigate results of both groups using an indicator. All students' performance ratio (refer to section) in both the previous cohort and current group were derived and produced the following average performance ratios: Average Performance Ratio of Group A (the flipped classroom group) = 0.605 Average Performance Ratio of Group B (the traditional lecture group) = 0.941.

These average performance ratios convey that students from Group B (the no flipped classroom group) performed better on the portion of partial fraction questions in comparison to Group A whose average performance ratio was significantly lower. Fig. 6 shows on average Group B (No flipped classroom) had a higher level of success in the partial fraction question compared to the rest of the examinable questions. This is displayed in the figure below.

The figure shows that a majority of students from Group B had a higher performance ratio and a steady set of outcomes throughout the group. The analysis of these results indicate that throughout the spectrum, the performance ratio values of students learning partial fractions in Group B were always higher than those of the students in Group A, as can be seen by the upward trend in Fig. 6.

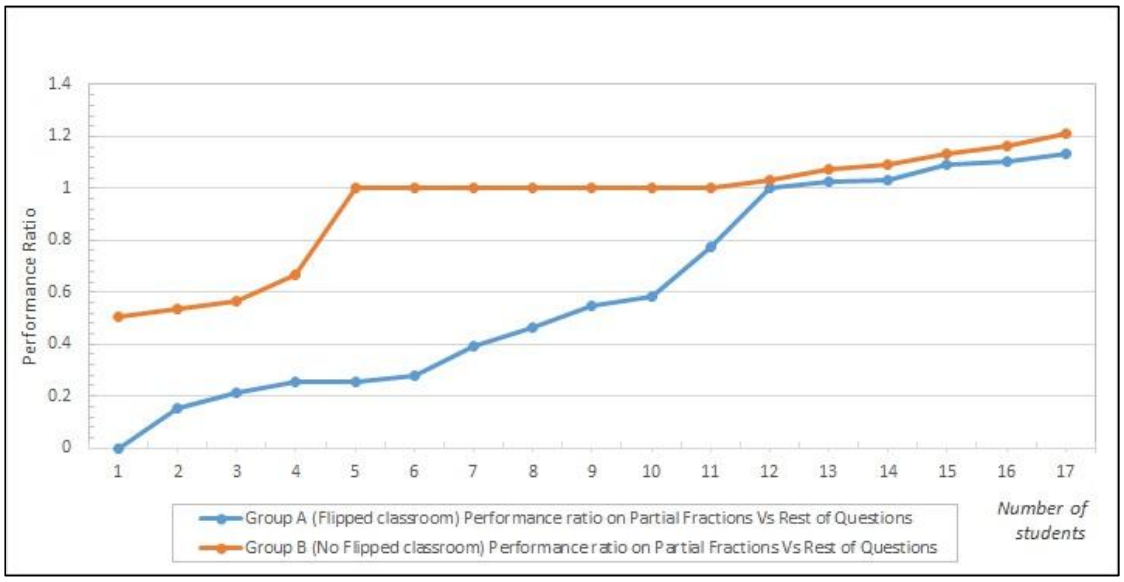

Fig. 6. Group A and Group B class performance ratio of partial fraction questions compared to the rest of the examinable questions against the respective number of students

In the published research of flipped classrooms, only a few studies, [19,20] reported that students in traditional classroom performed significantly better than the students in its flipped counterpart. Perhaps students consider learning introductory mathematics to be easier in the traditional lecture style because their classroom experience is more passive and because of increased contact time with their instructor, a viewpoint supported also by Gundlach et al, 2015, whereby the subject of their investigation is an Introductory Statistics course [21]. When used effectively, traditional course delivery methods can provide an efficient way to teach students an introductory level course and these may also be well-suited for introverted, academically weaker students, students who do not prepare reliably for class sessions, or those who are not willing to participate fully in class discussions [21,22].

Moreover, for some students where the flipped class structure represents unfamiliar territory, a flipped classroom model may not possess a positive effect on student performance [19]. In this study, the investigation was centred on a foundation level Science, Technology, Engineering, and Mathematics (STEM) related course, and for this reason, it can be expected that the students participating in this intervention, were not experienced with the flipped approach but in addition did not have the motivation or aptitudes to prepare beforehand independently the partial fraction topic compared to students of other levels of a degree that might be more equipped to handle this form of learning. Perhaps students midway through their studies are the most amenable to active learning applying their existing knowledge instead of passively listening to lectures.

Fig. 7 shows the percentage of students who performed better in the partial fractions question compared to the rest of the questions in the assessment. The results show that students in group A performed better in the rest of the questions, where these topics were taught in the traditional manner, and not so well for the topic where flipped classroom was introduced. Recall that a performance ratio greater than 1.0 indicates a better performance in the partial fraction section of the assessment. Therefore, when students worked under the flipped class arrangement, 35% of the class performed better in the partial fraction question relative to the rest of the questions in the assessment, whilst approximately 77% of the class achieved better marks on this topic under the traditional lecture environment. These percentages highlight that student performance under the flipped classroom model did not produce effectiveness in teaching mathematics, a deduction which is in coherence that a flipped classroom technique might not work successfully for all disciplines.

The "effect size" allowed a scientific approach into interpreting the effect of the flipped classroom model in students' performance. The effect size (d index) was important in interpreting the effectiveness of the study through using the difference in average performance ratios of Group A (Flipped classroom) and Group B (No flipped classroom), divided by the standard deviations of both group, in other words, the pooled standard deviation [18,23].

The results of the average performance ratio, standard deviations and d-index (Effect size) are summarised below.

The analysis of these results produced a large effect size of value 1.082 meaning that the two groups' means differ by approximately one standard deviation. Specifically, the difference between these two groups is big enough and consistent enough to be significant. These findings reinforce the validity and the importance of this investigation and how this paper enhances and adds to current literature concerning the learning value of the flipped classroom in introductory university level mathematics.

4.3 Discussion of Qualitative Analysis: Survey

This paper also aimed to assess how the flipped classroom model could improve engagement and learning in the classroom. Part of the investigation included students' responses in a survey using a Likert scale based on their experience in the flipped classroom model that they just had. This was used to analyse the 'engagement' aspect of the investigation and to understand how the flipped classroom model improved engagement and satisfaction in the classroom.

The first two questions of the survey aimed to investigate students' experience on engagement based on the flipped classroom model. In the first question included in Fig. 3, the statement "The flipped classroom is more engaging than the traditional classroom" was used to determine how engaging students found the flipped

classroom as opposed to the traditional classroom. Results showed that a minority had agreed with the statement with only 12 percent choosing "agree" and the majority either choosing "disagree" or "strongly disagree". This shows that when it came to a comparison between the two teaching techniques, students preferred the traditional classroom structure in terms of engagement. In addition, from the bar chart in Fig. 3 representing the statement "The flipped classroom creates more opportunities for me to engage with other students", this generated a wider variety of responses with equal amounts of students falling on either side of the Likert scale with a total of 41 percent choosing either "agree" or "strongly disagree" and a total of 41 percent also choosing either "disagree" or "strongly disagree". However, while the results were generally equal, most students (35 percent) agreed that the flipped classroom created more opportunities of engagement with other students.

These questions show that students found aspects of the flipped classroom to be engaging as it created opportunities for them to engage with other students and while for some students it wasn't more engaging than the traditional classroom, it had improved engagement for others. Other questions focused on the resources used and general experiences of the model by students. Fig. 3 also evidenced that 65 percent of students had claimed to have used the resources provided at home. Students who were more successful could have utilised the resources more. Questions 4 and 5 focused on students' experience of using videos in their learning. Results show that students appreciated using videos to learn new content with Fig. 4 showing that 59 percent of the students agreed with the statement "I liked watching videos to learn new content" allowing for the deduction that students liked certain aspects of the flipped classroom such as incorporating active learning techniques.

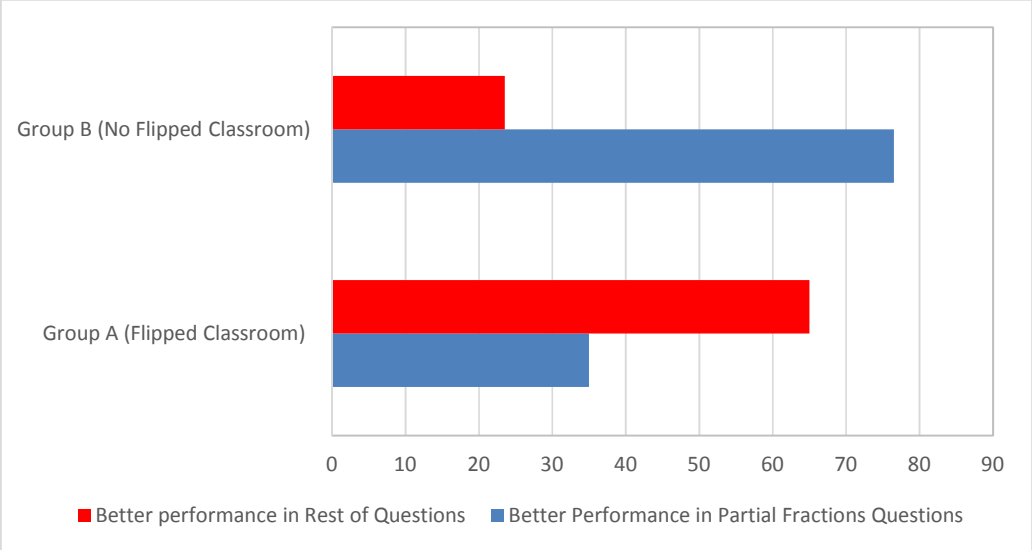

Fig. 7. Percentage of students performing better in partial fractions questions vs. the rest of the questions

Table 1. Summary of significant statistical metrics of the data

Group A (Flipped Classroom) Average Performance Ratio	0.605
Group B (No Flipped Classroom) Average Performance Ratio	0.941
Group A Standard Deviation	0.392
Group B Standard Deviation	0.224
Group A Class Size	17
Group B Class Size	17
Pooled Standard Deviation	0.3096
d - index (Effect Size)	1.082

However, results show that students ultimately preferred the traditional lecture over learning content using a video. This demonstrates that the videos were beneficial to some extent and experiences were independent upon each student. They also found the idea of using lesson time to answer questions beneficial showing that while there wasn't a strong preference to the flipped classroom in general, they still benefitted from the experience.

Fig. 5 showed that no students disagreed in any form with the idea of using lesson time to answer questions. Illustrating that while the traditional lecture is preferred over the flipped model, students would benefit in having certain features of the flipped classroom model incorporated into their lessons as evident in the results. The results displayed positive responses to the use of videos in the classroom after three weeks of applying this technique. Question 12 represented by Fig. 5 showed that a high number of students found the videos used to teach partial fractions difficult to understand. While students advocated for the use of videos to learn the content, the video used during the investigation was not helpful. A majority of students also felt that they would recommend the flipped classroom model to a peer showing that most students viewed the experience positively.

The analysis on engagement of the flipped classroom only focuses on students' experiences implementing the flipped approach for a duration of three weeks. In comparison, the traditional lecture has been used throughout the module in previous lectures. This means students need more time to adjust to the new model. It is difficult to determine how effectively students used their resources at home as the flipped model requires students to work independently. Learning content at home also requires adjustment from the usual style of learning but yet individuals study with a variety of resources at their own pace. This allowed them to revisit parts they did not understand and pause/play videos when they deemed necessary, a flexibility that could not be achieved in the classroom as lecturers do not have time to numerously revisit sections at the request of students.

The model also created a student-centred environment as they were able to receive support on the questions in class as opposed to the lack of support they had achieved when attempting homework outside the classroom. The model focused on the preferences of the individual allowing exploration when learning the topic of 'partial fractions' as it allowed students to access their preferred resources. The use of videos also allowed interactive learning which results showed students found accommodating.

While the investigation was effective and had many strengths, there are limitations that need considering. The sample size used can be considered small hence not a true representative of a larger group and further limited to an analytical mathematics course. The technique used also required students to study at home independently. While most students had claimed to have used the resources provided at home, there was difficulty in monitoring efforts outside the classroom making it impossible to know how each student approached the learning. This means that it would be difficult to conclude whether failure attempts on the exercises was due to lack of understanding or students' failure to work adequately at home. Learning new content requires focus which could have been difficult for some students as they work better in a classroom environment and not all students have the facilities needed to study at home such as technology or internet. The method ignores students' inability to accessing and being able to use technology efficiently.

Students' response was collected using a Likert scale, a universal method for survey collection which is therefore easily interpreted. The Likert scale allows a quantifiable set of results. The Likert scale also allows the respondent flexibility as it does not force them to give an opinion by providing the option of "Don't Know". While the Likert scale is advantageous, there are also disadvantages that must be taken into consideration. While attitudes and opinions of individuals are multi-dimensional, the Likert scale is unidimensional and only allows a set of choices where the space between choices is difficult to determine. It is also unlikely that responses given are true in its nature, with many people avoiding the extreme sides of the choices due to the implications surrounding "extremists" or even choosing to have no opinion on the matter.

4.4 Discussion of Quantitative Analysis: Performance Ratio and d-Index (Effect Size)

A performance ratio of higher than 1 suggests that students performed better in the partial fraction question compared to the rest of the

questions in the assignment. The results shown in Fig. 6 demonstrate that students from the group with no flipped classroom intervention, Group B, performed better on the portion of the assessment with the partial fraction questions compared that of Group A, the experimental class group. This achieves the result that students using the traditional lecture had a better understanding of the Partial fraction topic relative to the rest of the examinable questions.

This is further emphasised in the depiction of Fig. 7 that portrays no positive effect on student's performance under the flipped class arrangement for this module, a deduction which is in consistency with the view that the flipped classroom technique might not work successfully for all disciplines.

To further understand the impact of the flipped classroom model in terms of performance, the "effect size" was calibrated using statistical measures obtained during the investigation. Table 1. tabulates the "effect size", calculated to be 1.082 whereby an "effect size" of higher than 0.8 is considered to be strong [18,23]. This large index value means that the difference between the traditional group and the flipped class group, is big enough and substantial for the findings of this paper. Such statistical analysis lets for a more scientific approach when answering the research question "Does the flipped classroom model work in a mathematics classroom?" allowing for a response contributing to current literature, that learning foundation level mathematics at university under a flipped classroom regime might not provide enhancement in the students learning experience.

The use of an assessment to further analyse students' understanding meant that both groups were given equal opportunities to perform. Assignments set had similar questions and similar conditions. The work assigned was set to be completed at home which means students all worked outside the classroom environment. Employing formal assignments as an assessment tool, was beneficial as it made sure all students participate and attempt to increase their efforts as it contributed to final grades. Both groups consisted of a sample of 17 students, of which they possessed the same educational background, permitting for the comparison amongst groups to be fair.

However, some limitations of using assignments to analyse students' performance, is the difficulty of monitoring how students work whether independently and what resources they accessed in this time.

5. CONCLUSION

This investigation set out to find if the flipped classroom model could improve engagement and learning in a university foundation level Analytical mathematics module and the impact that an active learning technique could have on students' performance.

Research into learning techniques used in the classroom, show that there is an increase in the amount of methods used in teaching to improve student satisfaction and engagement in the classroom. In particular, there has been an increase in active and interactive learning used by educators in the classroom and a deviation away from the traditional styles of instruction-based learning such as lectures. The decline in passive approaches has achieved positive results and had a positive impact on student results.

This paper focuses on the use of the flipped classroom model and if it can also be seen as an effective teaching method in Mathematics. While it has proven success in a numerous amount of studies into different subject areas and different levels of education, there is a limited amount of research in flipped classroom in higher education Mathematics. This investigation aims to identify the level of engagement and improvement of student satisfaction as a result of implementing the flipped classroom model in an analytical mathematics module. Research has identified that some strengths of the flipped classroom include its flexibility and its ability to provide student with their own time to efficiently comprehend content, leaving class time to become more interactive and engaging for students.

The study identified whether these strengths and weakness are persistent when applying the model to a foundation year analytical mathematics module. In terms of the 'learning' aspect of the study, observation of results showed that performance decreased significantly when the flipped classroom model was introduced. It is possible to say that the flipped classroom model decreased success in students' performance as demonstrated in the

assignments. Results show that Group B (No flipped classroom) had performed better in the partial fraction section of the assignment relative to the rest of the questions entailed within. This suggests that the flipped classroom model failed to improve 'learning'.

On the other hand, with regard to 'engagement' there is evidence to suggest that the flipped classroom model improved engagement as a high number of students highlighted positive aspects of the flipped model. Students replied that the flipped classroom was not as engaging as the traditional lecture but certainly increased on engagement between peers. Features such as the use of videos and answering questions in class time were appreciated by students. Observations also showed that students suggested incorporating positives of the flipped classroom model into their traditional lectures by adapting to include the suggested aspects.

This paper contributes to current literature by examining the effectiveness of the flipped classroom approach and if it improves engagement. There is a limited amount of research in existence that assesses the potential effects on student learning as a result of flipped learning in STEM disciplines [24].

Most published work aims to investigate the effectiveness of the flipped classroom for an improvement in learning, but further research is needed to see how engagement in the flipped classroom is tackled and enhanced. This investigation highlights the increase in engagement opportunities as a result of the flipped classroom and the impact this has on learning experiences. There is a positive perception on the flipped classroom model in terms of students' engagement and new experiences it introduces into the classroom. The model inspires students to pursue their learning and improve engagement in their subjects [25].

To further reach a conclusion as to whether the flipped classroom model improves engagement and learning, the initially stated research questions need to be addressed. The first question "Does the flipped classroom model work?" was broad and subject to interpretation. Considering all aspects of the investigation such as the quantitative and qualitative analysis, it is deduced that to some extent the flipped classroom model was effective as it improved engagement amongst peers and had positive attributes. Based on better student performance, the flipped classroom did not seem to provide positive effects compared to the traditional way of learning, and hence it can be proposed that this approach might not work for all disciplines.

The research question "How effective is the flipped classroom in improving student satisfaction" aimed to understand students' opinions on the model. This was addressed through the qualitative aspect of the research. The responses to the survey show both positive and negative experiences towards the flipped classroom model. Many students were in support of attributes such as using class time to answer questions and found this beneficial. Whilst most students felt that the traditional classroom was more engaging than the flipped classroom model, students still liked factors of the flipped classroom and the student-centred approach that the flipped classroom holds.

6. LIMITATIONS

Many researchers and instructors from different areas have published work that favors the flipped classroom; going as far as saying students are performing better as a result of flipped learning [26]. The results from this investigation disprove this idea as it shows that as a result of introducing the flipped classroom model, students' performance decreased. The use of non interactive videos to learn leads to severe inactivity and can prohibit the improvement of learning effectiveness [27].

This paper provides further data on success in learning in the flipped classroom and input on engagement in the flipped classroom. Current research into the flipped classroom in mathematics focuses on the education provided in school and in the higher education sector in other countries. There is less research on the flipped classroom in mathematics at UK universities, as most research is based on either school education systems or universities abroad.

This investigation allows for a new perspective into the flipped classroom in UK universities and the impact it has on education. Previous data highlights the benefits of the flipped classroom in mathematics. Nevertheless, this investigation is important as it produces novel findings through its focus on the engagement in the flipped classroom and lack of success when applying the flipped model in a foundation level mathematics classroom. This extends and adds a new scope of literature into a different

perspective of the flipped classroom in mathematics.

The introduction of the flipped classroom leads a high number of strengths. The most recognised strengths of the flipped classroom is the flexibility it allows for students. The introduction of pre-recorded videos used in the flipped classroom allows students to control their learning as they are able to repeatedly access content and spend as much time as they deem necessary [28]. Accessibility to resources outside the classroom means students can view content in their own time [29].

The most successful results from the flipped classroom is from students who utilised a variety or resources and content online. In order for this success to take place students must have access to technology outside the institution [30]. This success is dependent on students' ability to access technology. This ignores the fact that not all students have the same access. While there are many strengths to be considered, there are also limitations to be thought of. When analysing the lower performance of students in the flipped classroom environment, it must also be noted that the quality of the support material used must be borne in mind. Further, students may not comprehend the information they are expected to learn at home and may also access the content while many other distractions take place. In the classroom, the educators are able to respond to such issues [31]. Other limitations can be the inability to monitor students' performance outside the classroom.

This paper used a relatively small sample size for both Group A and B which means that the results collected cannot be generalised to a larger population. Another significant limitation for this investigation is the length of time students used the flipped classroom. The model was undertaken for three weeks which means students were not given enough opportunity to adapt to the new model and observe the long-term impact that it could have on their performance.

7. FUTURE WORK

The effectiveness in teaching and learning of the flipped model is difficult to evaluate as it is harder to control external factors such as what resources students use and the efforts they put in outside the classroom. It is possible that student success and attitudes in the flipped section will improve as flipped courses increase in number at any institution, but still for some students, the flipped class structure represents unfamiliar territory.

The author of this investigation invites researchers to further investigate and enhance findings through an increase of sample size and the duration of the intervention. Modern technology encourages the use of active learning and employers are looking for employees who are able to work independently with a range of opportunities that encourage working from home and flexibly.

The flipped classroom model helps to prepare students for this work force enriching them with the experience needed. Future investigations into the flipped classroom should aim to focus on learning interactions as a response to the flipped classroom over a certain period of time. For instance, research can be conducted on the use of the flipped classroom structure for an entire module.

The flipped classroom model actively allows students to experience benefits of modern education and technology, creating a pathway for further active learning techniques. Reflecting on students' experiences and adapting learning techniques to match society and students' abilities allows for a positive reflection into the system used to deliver education.

CONSENT AND ETHICAL APPROVAL

As per university standard guideline participant consent and ethical approval has been collected and preserved by the authors.

ACKNOWLEDGEMENTS

The author would like to thank student, Ms. Fatma Ibrahim who assisted with the conducting of the investigation.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

Survey

	Strongly agree	Agree	Not sure	Disagree	Strongly disagree
The flipped classroom is more engaging than the traditional classroom.					
The flipped classroom creates opportunities for me to engage with other students.					
I used all the resources provided to study at home.					
I liked watching videos to learn the content.					
I would rather watch a traditional lecture than a lesson video.					
Using lesson time to answer questions is beneficial.					
I would recommend the flipped classroom to a friend.					
Learning to use the flipped classroom will benefit me in my future education.					
I am more motivated to learn maths in the flipped classroom.					
I struggled with learning content outside the classroom.					
The classroom should be only for learning new content.					
The videos used were easy to understand.					

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/64155