



Research on Geometric Intuition of Junior High School Students in China

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

Recently, geometric intuition, as one of the core literacies of mathematics in Junior high schools, has received widespread attention. Many scholars have researched this area in China, but it lacks literature that provides a complete overview and collation of geometric intuition. We searched with the keyword geometric intuition through CNKI, and carefully read and categorised the opinions and research approaches of existing research, and draw the following conclusions: (1) Current research in China mainly focuses on four aspects: the connotation and expression of geometric intuition, the value of geometric intuition, the measurement and current situation of geometric intuition, and the cultivation strategies of geometric intuition. (2) There are some shortcomings in previous studies. For example, the methodology is monotonous and the content of geometric intuition require further expanding. (3) There are some gaps in the research on this topic. For example, the connotation of geometric intuition in the new era needs to be explored in-depth. Additionally, there is a lack of operational strategies, which needs to be verified through practical teaching. Therefore, it is necessary to improve the research methodology and expand the scope of research on geometric intuition in order to make this research more comprehensive, systematic, and in-depth.

Keywords: *Geometric intuition; core literacy in mathematics; junior high school; geometric intuitive literacy.*

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1. INTRODUCTION

Geometric intuition is one of the core literacies of mathematics in compulsory education proposed in *The Compulsory Education Mathematics Curriculum Standards (2022 Edition)*. It helps to grasp the essence of problems and clarify the path of thinking. Teachers need to use geometric visualization to make implicit mathematical experiences explicit. Based on this explicit "evidence". Teachers can embody, regulate and enhance mathematical experiences, thus achieving an effective accumulation of mathematical experiences [1]. Therefore, it is important to study geometric intuition in face of the promotion of new curriculum reform in mathematics. By analyzing the existing literature, it is easy to find that research results on geometric intuition in Junior high school are plentiful, but there is rarely a more complete overview of the kinds of literature on this topic in China. Therefore, we intend to review the existing literature and systematically analyze the current status of research in China on geometric intuition at the junior high school level, to provide more perspectives for future research on geometric intuition. More importantly, by identifying the shortcomings and gaps in the literature, scholars will be motivated to conduct in-depth research and ultimately promote the cultivation of geometric intuition literacy among junior high school students.

The research questions of this paper are as follows:

- (1) What are the aspects of current research on geometric intuition at the junior secondary level in China?
- (2) What are the gaps and lacunae in the current research on geometric intuition at the junior secondary level?

2. METHODOLOGY

2.1 Data Sources

All literature covered in this paper comes from the CNKI (China National Knowledge Infrastructure) database. CNKI is the most authoritative literature search tool in Chinese academic journals, which contains all the contents of journals in China, including various subject areas. This database was chosen to ensure the persuasiveness and reliability of the study.

2.2 Data Collection

In order to avoid omissions in the literature, the search was conducted by entering the keywords 'geometric intuition' and 'geometric intuitive literacy', arranged in ascending order of publication time. However, after browsing through the results, it is found that there are different levels of research on geometric intuition in China, covering various fields such as higher education, primary education, and vocational education. Although a large amount of literature is searched, not much research is conducted on geometric intuition at the junior secondary level. Therefore, for all the data retrieved, the study used the following criteria for screening: (1) only Chinese literature was selected; (2) literature on geometric intuition as an explicit object of study at the junior high school level. Through screening, 43 pieces of literature are finally selected.

2.3 Data Analyzing

After an initial glance at the selected literature, it is found that the depth and breadth of research are limited, so each article is read carefully and take notes to collate the content of each aspect covered in the article.

3. RESULTS

Based on a summary and analysis of the existing literature, our preliminary view is that current research in China is focused on the connotation and expression of geometric intuition, the value of geometric intuition, the assessment and current status of geometric intuition, and strategies for developing geometric intuition.

3.1 The Connotation and Expression of Geometric Intuition

3.1.1 The connotation of geometric intuition

Geometric intuition, as one of the core literacies in mathematics, has a rich connotation. *The Compulsory Education Mathematics Curriculum Standards (2022 Edition)* clearly defines geometric intuition as the awareness and habit of using diagrams to describe and analyze problems. Some experts and scholars in China have explored the meaning of geometric intuition from different perspectives and have clarified their deeper understanding. Xu sums up his own experience and explains the term 'intuition' as the direct perception and understanding of relationships arising from experience,

observation, testing, or analogical association. The direct perception of quantitative relations through the visual relations of geometric figures seen or thought of can be called "geometric intuition" [2]. Chinese scholars are not unanimous in their understanding of geometric intuition, with some believing it to be an ability, and others believing it to be a way of thinking.

Some scholars argue that geometric intuition is an ability and explain the connotations of geometric intuition. Based on their interpretation of Geometric Intuition in *the Compulsory Education Mathematics Curriculum Standards (2011 Edition)*, Kong and Shi consider that geometric intuition refers to the ability to directly perceive and grasp the object of study in mathematics as a whole with the help of visual relations of the geometric figures seen [3]. Cui, and Chen, et al, explain geometric intuition as follows: Geometric Intuition is essentially the ability to analyze, understand and solve mathematical problems by directly generating a perception of the quantitative relationships and other essential properties of the object of study with the help of figures [4].

Other scholars consider geometric intuition to be a way of thinking. For example, Jiang defines geometric intuition from a psychological perspective as a directly mental state of recognition or conjecture of objective things and their relationships by the human brain [5]. Xu explains geometric intuition in both a narrow sense and a broad sense. Geometric intuition in a narrow sense is the direct perception of images or concrete perceptual responses to relevant geometric knowledge in the teaching process or geometry materials, and is a method or means of problem-solving; geometric intuition in a broad sense is the mental state of conjecture or direct recognition of objective objects and their relationships by the human brain, and is a form of thinking [6].

To better understand the difference between geometric intuition and other concepts, Li, and Su, et al, in their review of geometric intuition identify it with concepts related to spatial concepts, intuitive geometry, and number-form combination [7,8].

The above discussion of the connotations of geometric intuition shows that although the concept of geometric intuition is rich and difficult to grasp, there are some agreements among

researchers regarding the use of geometric figures to describe, analyze and solve problems. However, scholars are not unanimous in their understanding of the properties of geometric intuition, whether it is an ability, a way of thinking, or both; nor are they unanimous in their understanding of what geometric intuition specifically is, all of which need to be further explored.

3.1.2 The expression of geometric intuition

After reading and collating pieces of literature, it is found that there are not many studies on the manifestations of geometric intuition, and the following are typical views. Kong and Shi, in conjunction with the actual mathematics of primary and secondary schools, believe that geometric intuition is manifested in four specific forms of expression: physical intuition, parsimonious symbolic intuition, graphical intuition, and substitution intuition [3]. Zhu divided geometric intuition into intuitive insight, intuitive imagination, and intuitive construction [9]. Gu and Zhang found that the expressions of geometric intuition are intuitive representation, intuitive analysis, intuitive explanation, and intuitive discovery through the analysis of junior high school mathematics textbooks [10].

Most experts and scholars currently agree with Kong and Shi in their understanding of geometric intuition in China.

3.2 The Value of Geometric Intuition

The value of geometric intuition as one of the core literacies in mathematics is self-evident. Experts and scholars have explained its value from a variety of perspectives, including mathematical content and pedagogical implementation.

Many researchers elaborate on the value of geometric intuition in the context of specific mathematical content in junior high school. Ye argues that geometric intuition has a guiding, bridging, and catalytic role in mathematics [11]. Zhang considers the value of geometric intuition as a combination of numbers and shapes, geometric transformations, reasonable reasoning, and abstract concreteness [12]. In their study of distance problems, Geng and Wu found that the value of geometric intuition is that it helps students to understand mathematics intuitively and plays an important role in the learning process of mathematics [13].

Other scholars have analyzed the value of geometric intuition in teaching and learning activities from a pedagogical perspective. From the perspective of using geometric intuition as a teaching tool, Wang and Pan believe that geometric intuition can be used to make implicit mathematical activity experience visible, to materialize, regulate, enhance, and accumulate mathematical activity experience [1]. Yang, Ren, and Xu summarise the educational value of geometric intuition in terms of developing students' abilities, mainly in terms of their observations, spatial imagination, reasoning, and creative thinking [14,6]. Through teaching experiments, Zheng discovered that integrating geometric intuition into the teaching of mathematical application problems can enable students to describe and analyze mathematical problems more intuitively, find ideas for solving them and answer them in different ways, improve students' problem-solving ability, and thus improve teachers' teaching efficiency and teaching quality of mathematical application problems [15].

Gu and Zhang analyze the educational value of geometric intuition from both epistemological and methodological perspectives: in terms of epistemology, they believe that concepts are "dual" in nature, having both "numerical and formal characteristics", and that only by understanding them from both perspectives can they be well understood and their essential meaning grasped; in terms of methodology, they believe that geometric intuition is one of the ways of thinking and problem solving, which not only helps to explore ideas for problem-solving, but also allows one to gain an intuitive understanding of mathematics and grasp the essence of problems [10].

The above analysis indicates that geometric intuition has both epistemological and methodological value. It not only promotes students' learning of mathematical content but also improves the effectiveness and quality of teachers' teaching. However, the value of geometric intuition in the context of the information age needs to be explored in depth.

3.3 The Measurement and Status of Geometric Intuition

3.3.1 The measurement of geometric intuition

An analysis of the existing literature reveals that there is currently no unified system for assessing

geometric intuition in China. Experts and scholars have constructed their framework of assessment either from the connotations of geometric intuition or with the help of relevant theories.

Constructing assessment frameworks from the connotations of geometric intuition mostly establishes frameworks based on in-depth, actionable interpretations of the connotations. Wei constructs a framework for describing problems with the help of graphs, analyzing, understanding, and solving problems [16]. Xu establishes a framework for measuring three levels of understanding shapes and describing and solving problems with the help of shapes (Xun,2022).

Scholars have drawn on a variety of theories in constructing their assessment frameworks. For example, Tang and Fu drew on the research of Dutch scholars, the Van Seel couple, on the development of geometric thinking, and constructed a framework for measuring geometric intuition in compulsory education from four levels: perception, comprehension, grasping and reasoning, and provided descriptions and case studies for each level [17]. Xu combines mathematics curriculum standards and the analysis of Chinese examination questions and textbooks to prepare a test paper on the level of geometric intuition literacy of junior secondary school students and to classify the level of students' geometric intuition literacy with SOLO classification theory (Xu,2020).

Although many researchers have constructed assessments of geometric intuitive literacy, they have not been able to form a unified, academically recognized, and efficient assessment system, and there is a need for greater uniformity in the construction of assessment systems in the future.

3.3.2 The current state of geometric intuition

The study of the current state of each literacy is of great importance for monitoring and improving teaching and learning. Therefore, many researchers analyze the level of development and the actual situation of students' geometric intuitive literacy.

Research has shown that there are differences in the geometric intuition of junior secondary students. In an investigation of the mathematical abilities of first-year junior school students, Chen,

and Zhang, et al, find that among the eight mathematical ability indicators, those with lower scores are "spatial concepts" and "geometric intuition", and the standard deviation values are larger, indicating that students' cognitive differences are also larger [18]. Xu, and Cao, et al, look at the level of eighth-grade students' geometric intuitive literacy from a test paper of the unit, and the survey shows that the students' level of this literacy is uneven [19].

The majority of students met the requirements for geometric intuition in the curriculum standards. The relevant studies are shown in the Table 1.

The above analysis reveals that there is a wealth of research on the current state of geometric intuition, mostly using questionnaires and interviews. Due to the inconsistency of the assessment systems and the different samples selected, researchers have obtained different profiles of geometric intuition, but there is agreement that there are no significant differences between men and women and that there is a consensus that there is a weakness in the area of graphical problem-solving.

3.4 Strategies for Developing Geometric Intuitive Literacy

3.4.1 Textbooks

Textbooks are a major forum for developing intuitive literacy in geometry and have received a great deal of attention from the analysis of their content to the development of teaching materials.

There is a lot of content in different areas of mathematics that involves geometric intuition. For example, Zhao and Wang analyze some cases of Number and Algebra in junior high school textbooks. Although much of the content of "Number and Algebra" is abstract and non-intuitive, describing and representing the content of Number and Algebra with the help of graphs can help students to build up a visual image of the content in their minds and promote their understanding of the internal connections [22].

The textbooks are written with certain requirements for geometric intuition. In a specific analysis of the content of geometric intuition in junior high school textbooks, Bai holds that the textbooks are written with more emphasis on the development of students' "geometric intuition" in the seventh and first semester of the eighth grade; the requirements for "geometric intuition" gradually increase as the grade level increases [23]. Gu et al. analyzed the clues of textbooks and concluded that geometric intuition as a core literacy is one of the dark lines of textbook structure and the design of geometric intuition in junior high school mathematics textbooks should follow the principles of accuracy, wholeness, progressiveness, and reflectiveness [10].

Relevant research on the analysis of geometric intuition in the content of teaching materials helps teachers to make better use of the materials to develop students' geometric intuition literacy. The development of teaching materials is important, and certain principles need to be followed.

Table 1. Status of relevant studies

Research methodology	Research subjects	The current state of geometric intuition
Questionnaire method and interview method	Grade 8 student at a middle school	Most students' development of geometric intuition met the requirements of the national curriculum standards and that there were no gender differences in the development of geometric intuition between male and female students [20,16].
Questionnaire methodology	Secondary School Students from Ethnic Minority Areas in Yunnan Province	Middle school students' awareness and habits of representing, understanding, analyzing, and solving problems with the help of graphs were still weak, and their ability to use geometric intuition should be further improved [4].
Questionnaire methodology	Grade 8 students from East, Central and West China	The overall level of geometric intuition of eighth-grade students in China was average and there were significant differences in the performance of geometric intuition between students of different regions and academic levels [21].

3.4.2 Instructional design

Instructional design is an important part of teaching and learning, while researchers have focused on developing intuitive literacy in geometry as an aspect of instructional design.

Li discusses the teaching of junior high school algebra. He summarises the methods and approaches for teaching geometric intuition in junior high school algebra as mastering basic shapes and using them to construct complex shapes; guiding students to draw geometric intuition from "number" to "shape", and cultivating the habit of thinking about problems by drawing diagrams [24]. Based on the basic requirements of each specific aspect of junior high school mathematics teaching design, Zhou analyses three teaching cases to develop a way of thinking from "number" to "shape" [25].

Scholars have not designed much teaching on content related to geometric intuition, mostly through case studies that demonstrate methods of instructional design but lack general instructional design strategies.

3.4.3 Teaching implementation

The development of geometric intuitive literacy ultimately comes down to the basic activity of daily teaching. Research on teaching strategies initially begins with the analysis of specific case studies. Huang uses examples to illustrate the use of geometric intuition in secondary school mathematics teaching and how to apply technical treatments such as variation and positive transfer to shapes to better exploit the positive effects of intuition [26]. Jiang discusses how geometry teaching in rural junior secondary schools should link geometric intuition with inspirational teaching by using a junior secondary school classroom lesson on plane geometry as an example [27].

Song constructs a theoretical framework for developing geometric intuition based on the intuitive principles of mathematics classroom teaching. At the core of this framework are the three horizontal and five verticals in the middle. The cultivation of geometric intuition in junior high school students requires reliance on the theoretical foundation and the exploration of five measures for the cultivation of geometric intuition to help students understand concepts, train their mathematical learning skills and improve their problem-solving abilities when learning mathematics [28].

The researchers offer suggestions from different entry points for teachers. In teaching triangles, Chi argues that the cultivation of geometric intuition requires hands-on practice, infiltrates the integration of numbers and shapes, cultivates good learning habits of drawing, and provides examples of exploring the cultivation of geometric intuition in the review lesson of Isosceles Triangles [29]. Zhang and Pei propose strategies for cultivating geometric intuition based on an overview of primary and secondary school students' intuitive imaginative literacy from two paths: teaching and assessment [30]. He proposes to improve students' geometric intuition by gradually developing the habit of drawing diagrams and learning to understand mathematics from both the "number" and "shape" perspectives in teaching [31].

With the development of information technology, scholars have focused on the role of information technology in developing literacy. Li, Hu, and Zhou advocate the appropriate use of the convenient features of modern computer technology in teaching and learning to develop geometric intuitive literacy, such as the Super Geometry Drawing Board, and even provide examples of teaching [32-34].

As can be seen from the above analysis, scholars have made diverse pedagogical suggestions for the implementation of geometric intuition literacy, mainly in terms of the understanding, conversion and innovation of knowledge. For example, understanding of theorems and formulae; geometric intuition in algebraic teaching; and the development of geometric intuition with the integration of information technology. Only a few researchers have carried out experimental verification, such as Wang, who has conducted experiments to verify the effectiveness of the proposed teaching strategies [35]. At the same time, some of the suggestions are more theoretical and not very helpful to front-line teachers' teaching, which is to be improved in the follow-up.

4. DISCUSSION

In recent years, core literacy has become a high-frequency keyword widely used by scholars and has become a focal issue in educational research. Chinese scholars have conducted relatively fruitful research on geometric intuition. Since the introduction of geometric intuition in the curriculum standards, research on geometric intuition at the junior high school level has been

growing year by year. By combing through the literature, it is found that the research on geometric intuition is mainly focused on the definition of connotation, value mining, and cultivation strategies.

In light of the above connotations of geometric intuition, the understanding of geometric intuition has evolved from superficial to profound over a long period of development. Geometric intuition has evolved from 'implicit' in its gestation period, to 'explicit' in its infancy, to 'comprehensive' in its current maturity [36-38]. The academic community has analyzed the connotation of geometric intuition from different perspectives, such as psychology, philosophy, and mathematics. Previous studies have provided a certain degree of rigor and completeness to the discussion of its connotations, and the understanding of geometric intuition has reached a consensus on the use of geometric figures to describe, analyze and solve problems, which has undoubtedly laid a solid theoretical foundation for further research. However, scholars are not unanimous in their understanding of the properties of geometric intuition, whether it is an ability, a way of thinking, or both; nor are they unanimous in their understanding of what geometric intuition specifically is, all of which need to be further explored.

The value of geometric intuition, as described above, has been highlighted since *the Compulsory Mathematics Curriculum Standards (2011 Edition)* formally introduced geometric intuition as one of the core concepts. Geometric intuition has both epistemological and methodological value. The academic consensus is that geometric intuition is important and of great value. Scholars have analyzed its value in-depth, both epistemologically and methodologically, in the context of specific pedagogical content, making researchers and teachers aware of the value of geometric intuition. This lays a solid foundation for further research and teaching of geometric intuition.

From the above-mentioned current state of geometric intuition among junior high school students. There are many studies on the current state of geometric intuition, mostly investigative analyses of students' abilities. Researchers have constructed measurement frameworks and used literature, survey, and interview methods to conduct in-depth studies. However, due to the differences in assessment frameworks, levels of study, and focus of study, the conclusions drawn

from studies of junior high school students' geometric intuition are mixed and even contradictory. Therefore, it is necessary to adopt different research methods such as observation and experimental methods to find out the current situation. Researchers should find a more reliable and valid measurement method, establish a purposeful model, and develop more comprehensive questionnaires to conduct empirical research on geometric intuition for students at different stages.

From the above strategies for developing geometric intuition, scholars have made diverse suggestions for the implementation of geometric intuition literacy involved a variety of aspects, such as teaching materials, instructional design, and implementation. Previous studies have not examined teaching materials and instructional design in-depth, with only a few examples available for study. At the same time, research on the development of pedagogical proposals is more thorough and involves a variety of perspectives to develop a richer theory.

It can be seen that the previous research has been very extensive, but there are still some shortcomings. Firstly, regarding the research methods of previous researchers, most of them have adopted qualitative research methods and theoretical thinking, and in quantitative research, they have also adopted questionnaires and data analysis methods, etc. Not many studies have adopted experimental and observational methods. The research methods are rather limited, which reduces the persuasiveness and credibility of the research to a certain extent. In addition, in terms of research content, previous studies have not yet developed a comprehensive and unified framework for measuring geometric intuition, which makes it difficult to investigate the current situation of geometric intuition. Most of the strategies for developing geometric intuition are theoretical and somewhat broad, which does not easily translate to teachers' practical teaching and is difficult to implement. Therefore, there is a strong need to improve the research methodology, adopt a more scientific approach, develop a more unified framework for assessing geometric intuition, and further explore the development strategies from an operational perspective.

The above analysis shows that there are some gaps in previous research. For example, there is little literature on how the value of geometric intuition has changed and developed in the

information age, which needs to be explored in depth; there is no scientific and unified assessment system for geometric intuition in literacy, which needs to be explored in depth; strategies for the development of geometric intuition are lacking in actionable strategies and need to be tested through practical teaching. Therefore, it is necessary to expand the scope of research in the future to make this study more systematic and comprehensive.

5. CONCLUSION

It is of great value to review and summarise research on geometric intuition in junior high schools. Through reviewing, sorting, and analyzing previous research in China, the conclusions are drawn as follows:

- (1) Currently, the research on geometric intuition in junior high schools in China focuses on four areas: the connotation and presentation of geometric intuition, the value of geometric intuition, the measurement and current status of geometric intuition, and strategies for developing geometric intuition. The basic research questions have been covered by previous researchers, and there is already a certain degree of research foundation and depth.
- (2) There are shortcomings in the previous studies. For example, the research methods of previous generations are monotonous, mostly using theoretical and discursive methods, and other quantitative research methods, apart from questionnaires, are rarely involved; the research content also needs to be further expanded.
- (3) There are some gaps in the research on this topic. For example, there is a need to dig deeper into the connotation of geometric intuition in the new era; there is a lack of a unified and scientific evaluation system to monitor the current situation of students' geometric intuition; and finally, there is a lack of operational strategies for the cultivation of geometric intuition, which needs to be verified through practical teaching.

Therefore, there is a need to improve research methods based on existing research, adopt more scientific and varied research methods, expand the scope of research on geometric intuition, and to further deepen related research. It is also

necessary to develop a more unified framework for the assessment of geometric intuition and to further explore the development of strategies from the perspective of operational implementation, to make this research more comprehensive, systematic, and in-depth.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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